EQUATION OF STATE FOR LITHIUM IN SHOCK WAVES

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Summary. An equation of state in the form of an analytic function of the pressure on the specific volume and the internal energy is proposed for lithium in the bcc-solid and liquid phases. The principal Hugoniot adiabat is calculated for the metal and compared with available shock-wave data. The results of the calculations are in good agreement with the data over the whole range of kinematic and dynamic characteristics investigated. This equation of state can be used effectively in simulations of shock-wave processes in lithium at high pressures.

1 INTRODUCTION

Equations of state (EOSs) of materials are necessary in hydrodynamic simulations of physical processes at high energy densities [1–3]. Such processes take place at high-velocity impingement of bodies [4–11], interaction of intense laser [12–23] and particle beams [24–28] with a condensed medium, electrical explosion of conductors under the action of high power current pulses [29–36], etc. The adequacy of the simulation results is determined mainly by accuracy of thermodynamic description of the materials response upon the changes in surrounding conditions over a wide range of pressures and specific volumes [37, 38].

For wide-range EOS modeling, a semiempirical approach is commonly applied [1], where a functional form of a thermodynamic potential is chosen via theoretical considerations, while numerical coefficients in the form are determined using experimental data [39–41].

In the work, a semiempirical EOS for lithium is proposed. This metal is used as a coolant in power plants, especially in molten salt reactors. In particular, EOS for lithium is of interest for numerical modeling of different working regimes of such reactors.

Unlike more complex EOSs of Li [2, 42–45], a simple analytic function $P = P(V,E)$ [46–48] is adapted for the metal, where, P is the pressure, $V = \rho^{-1}$ is the specific volume, ρ is the density, E is the specific internal energy. Thermodynamic characteristics of lithium along the principal Hugoniot adiabat are calculated and compared with available data from shock-wave experiments [49–52].

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2 EOS MODEL

The EOS model is formulated in the general form as

$$P(V,E) = P_{\mathrm{c}}(V) + \frac{\Gamma(V,E)}{V}[E - E_{\mathrm{c}}(V)]. \tag{1}$$

Here, E_{c} and $P_{\mathrm{c}} = -\mathrm{d}E_{\mathrm{c}}/\mathrm{d}V$ are the cold components of internal energy and pressure at $T = 0$ given by polynomials [53–58]:

$$E_{\mathrm{c}}(V) = \frac{B_{0\mathrm{c}}V_{0\mathrm{c}}}{m-n}\left(\frac{\sigma_{\mathrm{c}}^{m}}{m} - \frac{\sigma_{\mathrm{c}}^{n}}{n}\right) + E_{\mathrm{sub}} \tag{2}$$

and

$$P_{\mathrm{c}}(V) = \frac{B_{0\mathrm{c}}\sigma_{\mathrm{c}}}{m-n}(\sigma_{\mathrm{c}}^{m} - \sigma_{\mathrm{c}}^{n}), \tag{3}$$

$\sigma_{\mathrm{c}} = V_{0\mathrm{c}}/V$; $V_{0\mathrm{c}}$ and $B_{0\mathrm{c}}$ are the specific volume and bulk modulus at $P = 0$ and $T = 0$. The quantity E_{sub} has meaning of the sublimation energy; it is determined by a normalization condition $E_{\mathrm{c}}(V_{0\mathrm{c}}) = 0$, which gives

$$E_{\mathrm{sub}} = \frac{B_{0\mathrm{c}}V_{0\mathrm{c}}}{mn}. \tag{4}$$

Parameters m and n are determined using shock-wave data for solid samples.

The coefficient Γ as a function of the volume and internal energy is defined analogously to caloric models [56–61] in the following form:

$$\Gamma(V,E) = \gamma_{\mathrm{i}} + \frac{\gamma_{\mathrm{c}}(V) - \gamma_{\mathrm{i}}}{1 + \sigma^{-2/3}[E - E_{\mathrm{c}}(V)]/E_{\mathrm{a}}}, \tag{5}$$

where

$$\gamma_{\mathrm{c}}(V) = 2/3 + (\gamma_{0\mathrm{c}} - 2/3)\frac{\sigma_{\mathrm{n}}^{2} + \ln^{2}\sigma_{\mathrm{m}}}{\sigma_{\mathrm{n}}^{2} + \ln^{2}(\sigma/\sigma_{\mathrm{m}})}, \tag{6}$$

$\sigma = V_0/V$; V_0 is the specific volume under normal conditions; $\gamma_{\mathrm{c}}(V)$ corresponds to the case of low thermal energies, and γ_{i} characterizes the region of highly-heated matter. The value of E_{a} defines the thermal energy of the transition of Γ from one limiting case to the other; it is determined from the results of shock-wave experiments at high pressures. From equations (1), (5) and (6), one can obtain a relation of the parameter $\gamma_{0\mathrm{c}}$ with values of the Grüneisen coefficient $\gamma = V(\partial P/\partial E)_V$, the internal energy and the specific volume under normal conditions (γ_0, E_0 and V_0, respectively):

$$\gamma_{0\mathrm{c}} = \gamma_{\mathrm{i}} + (\gamma_0 - \gamma_{\mathrm{i}})\left[1 + \frac{E_0 - E_{\mathrm{c}}(V_0)}{E_{\mathrm{a}}}\right]^2. \tag{7}$$

Therefore, the interpolational function (6) ensures validity of the condition $\gamma(V_0, E_0) = \gamma_0$, and gives the asymptotic value $\gamma_c = 2/3$ in the limiting cases of low and high compression ratios σ. The parameters σ_n and σ_m are determined from the requirement of optimum fit to experimental data on shock compressibility of a substance in question.

3 EOS FOR LITHIUM

Lithium in the solid phase under atmospheric pressure has a body-centered cubic (bcc) structure ($T > 75$ K) [62]; it melts at 454 K. Under static compression at room temperature, the bcc phase transforms at pressure 6.9 GPa [63] to the phase with a face-centered cubic (fcc) structure. At further increase of pressure at room temperature, more crystalline phases of lithium are observed [64–66].

Shock compressibility of lithium is studied with the use of traditional explosive systems up to 70 GPa [49–52]. Shock compression leads to increase of temperature and melting of the bcc phase [44].

In this work, the unified EOS for the bcc-solid and liquid phases of lithium is constructed. The EOS coefficients are as follows: $V_0 = 1.8868\ \mathrm{cm}^3/\mathrm{g}$, $V_{0c} = 1.8422\ \mathrm{cm}^3/\mathrm{g}$, $B_{0c} = 11.887$ GPa, $m = 0.67$, $n = 0.48$, $\sigma_m = 0.9$, $\sigma_n = 1$, $\gamma_{0c} = 0.6$, $\gamma_i = 0.45$ and $E_a = 5$ kJ/g.

Calculated principal Hugoniot adiabat of lithium is presented in figures 1–3 in comparison with experimental data [49–52]. Calculation of the Hugoniot adiabat is performed by solving the equation of energy conservation in the shock front [1]:

$$E = E_0 + \frac{1}{2}(P_0 + P)(V_0 - V), \tag{8}$$

where the left-hand side is closed by the EOS function $E = E(P, V)$. Equation (8) and the EOS determine the specific volume as a function of pressure along the Hugoniot adiabat for samples of initial density $\rho_0 = V_0^{-1}$. The shock (U_s) and particle (U_p) velocities are calculated using the equations of conservation of mass and momentum in the shock front [1]:

$$U_s = V_0\sqrt{(P - P_0)/(V_0 - V)}, \tag{9}$$

$$U_p = \sqrt{(P - P_0)(V_0 - V)}. \tag{10}$$

As one can see in figures 1–3, the obtained EOS provides for reliable description of thermodynamic properties of the metal over the entire range of shock and particle velocities, pressures and compression ratios investigated.

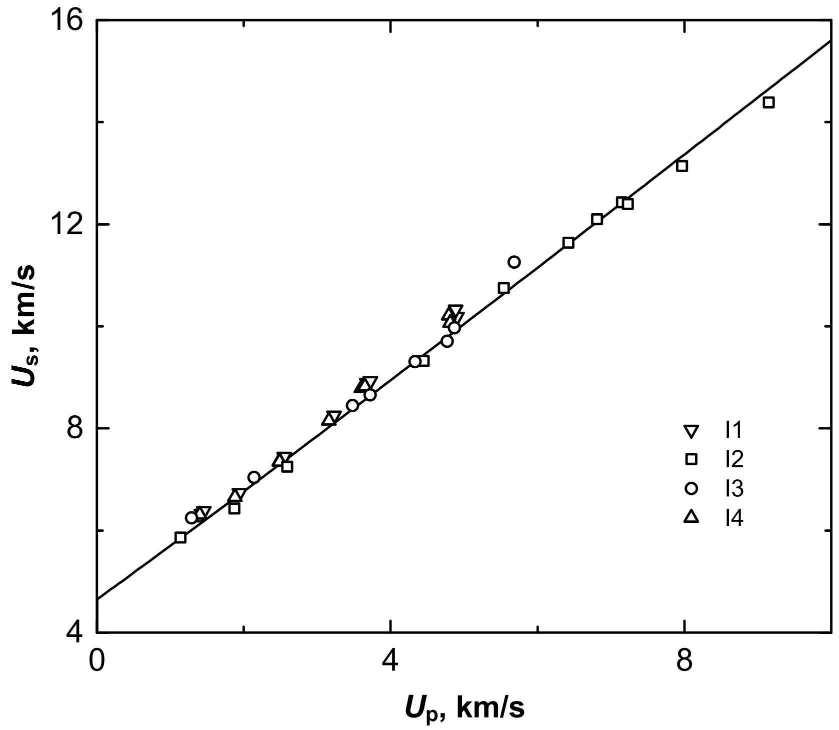

Figure 1. The principal Hugoniot adiabat of lithium: curve corresponds to the present calculations; markers—experimental data (I1—[49], I2—[50], I3—[51], I4—[52]).

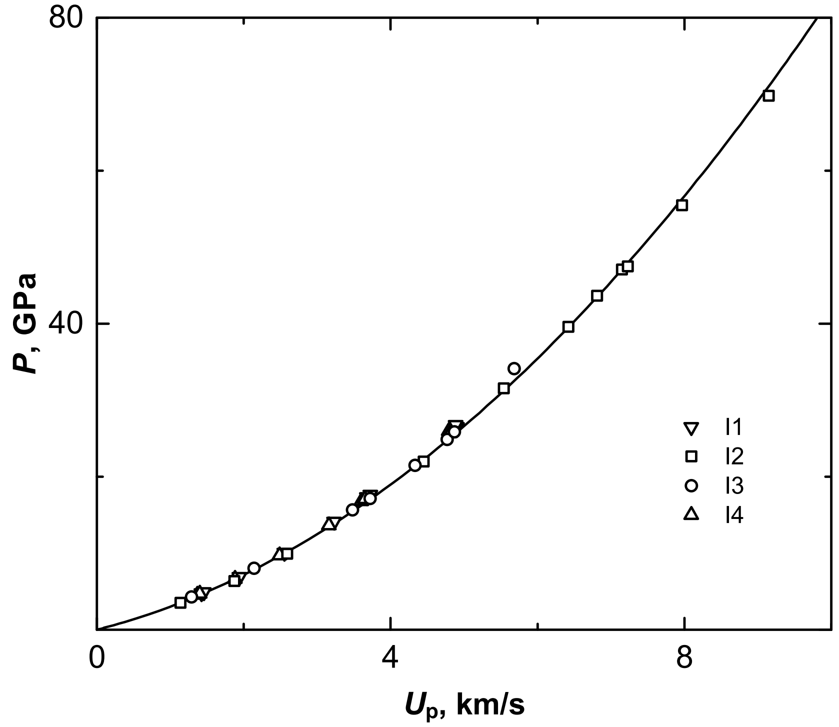

Figure 2. The principal Hugoniot adiabat of lithium: notations are analogous to figure 1.

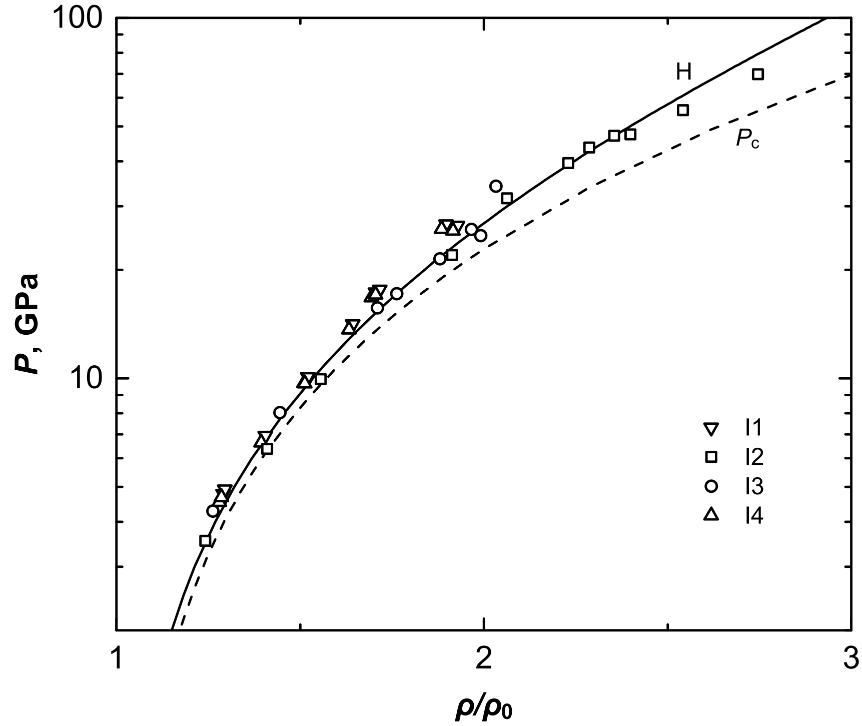

Figure 3. The cold curve (P_c) and the principal Hugoniot adiabat (H) of lithium: curves correspond to the present calculations; markers—experimental data (I1—[49], I2—[50], I3—[51], I4—[52]).

4 CONCLUSIONS

The EOS in the form of an analytic function $P = P(V,E)$ is created for lithium in the bcc-solid and liquid phases. This EOS agrees well with available shock-wave data; it can be used effectively in numerical simulations of dynamic processes in the metal at high pressures.

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REFERENCES

[1] Ya. B. Zel'dovich and Yu. P. Raizer, *Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena*, New York: Academic Press, (1967).
[2] A. V. Bushman, V. E. Fortov, G. I. Kanel', and A. L. Ni, *Intense Dynamic Loading of Condensed Matter*, Washington: Taylor & Francis, (1993).
[3] V. Fortov, *Thermodynamics and Equations of State for Matter: From Ideal Gas to Quark–Gluon Plasma*, Singapore: World Scientific Publishing, (2016).
[4] V. E. Fortov, V. V. Kim, I. V. Lomonosov, A. V. Matveichev, and A. V. Ostrik, "Numerical modeling of hypervelocity impacts", *Int. J. Impact Eng.* **33**(1–12), 244–253 (2006).

[5] M. E. Povarnitsyn, K. V. Khishchenko, and P. R. Levashov, “Hypervelocity impact modeling with different equations of state”, *Int. J. Impact Eng.* **33**(1–12), 625–633 (2006).
[6] L. V. Shurshalov, A. A. Charakhch’yan, and K. V. Khishchenko, “Numerous experiment on impact compression of a mixture of graphite and water”, *Combust., Explos. Shock Waves* **53**(4), 471–478 (2017).
[7] E. I. Kraus and I. I. Shabalin, “The tool for high-velocity interaction and damage of solids”, *Mathematica Montisnigri* **39**, 18–29 (2017).
[8] A. L. Coleman, R. Briggs, M. G. Gorman, S. Ali, A. Lazicki, D. C. Swift, P. G. Stubley, E. McBride, G. Collins, J. S. Wark, and M. I. McMahon, “Implementation of hydrodynamic simulation code in shock experiment design for alkali metals”, *J. Phys.: Conf. Ser.* **950**, 042037 (2017).
[9] A. E. Mayer and A. A. Ebel, “Shock-induced compaction of nanoparticle layers into nanostructured coating”, *J. Appl. Phys.* **122**(16), 165901 (2017).
[10] V. V. Pogorelko, V. S. Krasnikov, and A. E. Mayer, “High-speed collision of copper nanoparticles with aluminum surface: Inclined impact, interaction with roughness and multiple impact”, *Comput. Mater. Sci.* **142**, 108–121 (2018).
[11] K. V. Khishchenko, A. A. Charakhch’yan, and L. V. Shurshalov, “On a heat exchange problem under sharply changing external conditions”, *Comput. Math. Math. Phys.* **58**(2), 286–293 (2018).
[12] M. Povarnitsyn, K. Khishchenko, P. Levashov, A. Zakharenkov, J. Hermann, S. De Falco, and T. Itina, “Simulation of ultrashort double pulse laser ablation of metals”, *J. Optoelectron. Adv. Mater.* **12**(3), 674–676 (2010).
[13] M. E. Povarnitsyn, N. E. Andreev, P. R. Levashov, K. V. Khishchenko, and O. N. Rosmej, “Dynamics of thin metal foils irradiated by moderate-contrast high-intensity laser beams”, *Phys. Plasmas* **19**(2), 023110 (2012).
[14] S. A. Abrosimov, A. P. Bazhulin, V. V. Voronov, A. A. Geras’kin, I. K. Krasyuk, P. Pashinin, A. Semenov, I. A. Stuchebryukhov, K. V. Khishchenko, and V. E. Fortov, “Specific features of the behaviour of targets under negative pressures created by a picosecond laser pulse”, *Quantum Electron.* **43**(3), 246–251 (2013).
[15] N. A. Inogamov, V. V. Zhakhovsky, Y. V. Petrov, V. A. Khokhlov, S. I. Ashitkov, K. V. Khishchenko, K. P. Migdal, D. K. Ilnitsky, Y. N. Emirov, P. S. Komarov, V. V. Shepelev, C. W. Miller, I. I. Oleynik, M. B. Agranat, A. V. Andriyash, S. I. Anisimov, and V. E. Fortov, “Electron-ion relaxation, phase transitions, and surface nano-structuring produced by ultrashort laser pulses in metals”, *Contrib. Plasma Phys.* **53**(10), 796–810 (2013).
[16] P. V. Breslavskiy, A. V. Mazhukin, and O. N. Koroleva, “Simulation of the dynamics of plasma expansion, the formation and interaction of shock and heat waves in the gas at the nanosecond laser irradiation”, *Mathematica Montisnigri* **33**, 5–24 (2015).
[17] A. A. Samokhin, V. I. Mazhukin, A. V. Shapranov, M. M. Demin, and P. A. Pivovarov, “Continual and molecular-dynamic modeling of phase transitions during laser ablation”, *Mathematica Montisnigri* **33**, 25–42 (2015).
[18] A. A. Samokhin and P. A. Pivovarov, “On spinodal manifestation during fast heating and evaporation of thin liquid film”, *Mathematica Montisnigri* **33**, 125–128 (2015).
[19] N. E. Andreev, M. E. Povarnitsyn, M. E. Veysman, A. Ya. Faenov, P. R. Levashov, K. Khishchenko, T. A. Pikuz, A. I. Magunov, O. N. Rosmej, A. Blazevic, A. Pelka, G. Schaumann, M. Schollmeier, and M. Roth, “Interaction of annular-focused laser beams with solid targets”, *Laser Part. Beams* **33**(3), 541–550 (2015).
[20] I. K. Krasyuk, P. P. Pashinin, A. Yu. Semenov, K. V. Khishchenko, and V. E. Fortov, “Study of extreme states of matter at high energy densities and high strain rates with powerful lasers”, *Laser Phys.* **26**(9), 094001 (2016).
[21] V. I. Mazhukin, A. V. Shapranov, A. V. Mazhukin, and O. N. Koroleva, “Mathematical formulation of a kinetic version of Stefan problem for heterogeneous melting/crystallization of metals”, *Mathematica Montisnigri* **36**, 58–77 (2016).
[22] A. E. Zubko and A. A. Samokhin, “Modeling of thermoacoustic and evaporation pressure signals in absorbing liquids irradiated with nanosecond laser pulses”, *Mathematica Montisnigri* **36**, 78–85 (2016).

[23] A. A. Samokhin, S. I. Kudryashov, A. E. Zubko, and A. V. Sidorin, "Modelling of nanosecond laser ablation. Continual approach", *Mathematica Montisnigri* **37**, 76–90 (2016).
[24] A. A. Charakhch'yan and K. V. Khishchenko, "Plane thermonuclear detonation waves initiated by proton beams and quasi-one-dimensional model of fast ignition", *Laser Part. Beams* **33**(1), 65–80 (2015).
[25] S. F. Gnyusov, V. P. Rotshtein, A. E. Mayer, V. V. Rostov, A. V. Gunin, K. Khishchenko, and P. Levashov, "Simulation and experimental investigation of the spall fracture of 304L stainless steel irradiated by a nanosecond relativistic high-current electron beam", *Int. J. Fract.* **199**(1), 59–70 (2016).
[26] A. A. Frolova, K. V. Khishchenko, and A. A. Charakhch'yan, "Track method for the calculation of plasma heating by charged thermonuclear reaction products for axisymmetric flows", *Comput. Math. Math. Phys.* **56**(3), 437–449 (2016).
[27] M. E. Zhukovsky, M. B. Markov, S. V. Podolyako, I. A. Tarakanov, R. V. Uskov, A. M. Chlenov, and V. F. Zinchenko, "Researching the spectrum of bremsstrahlung generated by the RIUS-5 electron accelerator", *Mathematica Montisnigri* **35**, 54–67 (2016).
[28] K. K. Inozemtseva, M. B. Markov, and F. N. Voronin, "The electromagnetic and thermomechanical effects of electron beam on the solid barrier", *Mathematica Montisnigri* **39**, 79–100 (2017).
[29] A. G. Rousskikh, R. B. Baksht, S. A. Chaikovsky, A. V. Fedunin, K. V. Khishchenko, A. Yu. Labetsky, P. R. Levashov, A. V. Shishlov, and S. I. Tkachenko, "The effects of preheating of a fine tungsten wire and the polarity of a high-voltage electrode on the energy characteristics of an electrically exploded wire in vacuum", *IEEE Trans. Plasma Sci.* **34**(5), 2232–2238 (2006).
[30] E. V. Grabovskii, P. R. Levashov, G. M. Oleinik, C. L. Olson, P. V. Sasorov, V. P. Smirnov, S. I. Tkachenko, and K. V. Khishchenko, "Formation and dynamics of plasma layers formed on the foil surface under the action of a high-current pulse", *Plasma Phys. Rep.* **32**(9), 718–728 (2006).
[31] V. I. Oreshkin, K. V. Khishchenko, P. R. Levashov, A. G. Rousskikh, and S. A. Chaikovskii, "Strata formation at fast electrical explosion of cylindrical conductors", *High Temp.* **50**(5), 584–595 (2012).
[32] S. I. Tkachenko, V. A. Gasilov, A. Yu. Krukovskiy, O. G. Olkhovskaya, and I. P. Tsygvintsev, "Computational model and numerical analysis of a thin aluminum wire electric explosion", *Mathematica Montisnigri* **28**, 39–61 (2013).
[33] V. N. Senchenko, R. S. Belikov, and V. S. Popov, "Experimental investigation of refractory metals in the premelting region during fast heating", *J. Phys.: Conf. Ser.* **653**, 012100 (2015).
[34] V. N. Senchenko, R. S. Belikov, and V. S. Popov, "Experimental investigation of thermophysical properties of eutectic Mo–C, graphite and tantalum at high temperatures", *J. Phys.: Conf. Ser.* **774**, 012020 (2016).
[35] A. Rososhek, S. Efimov, M. Nitishinski, D. Yanuka, S. V. Tewari, V. Tz. Gurovich, K. Khishchenko, and Ya. E. Krasik, "Spherical wire arrays electrical explosion in water and glycerol", *Phys. Plasmas* **24**(12), 122705 (2017).
[36] V. N. Senchenko and R. S. Belikov, "Experimental investigation of density of pyrolytic graphite up to melting point", *J. Phys.: Conf. Ser.* **946**, 012105 (2018).
[37] S. I. Tkachenko, P. R. Levashov, and K. V. Khishchenko, "Analysis of electrical conductivity measurements in strongly coupled tungsten and aluminum plasmas", *Czech. J. Phys.* **56**, B419–B424 (2006).
[38] S. I. Tkachenko, P. R. Levashov, and K. V. Khishchenko, "The influence of an equation of state on the interpretation of electrical conductivity measurements in strongly coupled tungsten plasma", *J. Phys. A: Math. Gen.* **39**(23), 7597–7603 (2006).
[39] A. V. Bushman and V. E. Fortov, "Model equations of state", *Sov. Phys. Usp.* **26**(6), 465–496 (1983).
[40] V. E. Fortov and I. V. Lomonosov, "Ya. B. Zeldovich and equation of state problems for matter under extreme conditions", *Phys. Usp.* **57**(3), 219–233 (2014).
[41] I. V. Lomonosov and S. V. Fortova, "Wide-range semiempirical equations of state of matter for numerical simulation on high-energy processes", *High Temp.* **55**(4), 585–610 (2017).
[42] M. Ross, "Extension of liquid-metal theory to dense partially ionized plasmas", *Phys. Rev. B* **21**(8), 3140–3151 (1980).
[43] D. A. Young and M. Ross, "Theoretical high-pressure equations of state and phase diagrams of the alkali metals", *Phys. Rev. B* **29**(2), 682–691 (1984).

[44] K. V. Khishchenko, "Equations of state for two alkali metals at high temperatures", *J. Phys.: Conf. Ser.* **98**, 032023 (2008).

[45] D. K. Belashchenko, "Impact compression of alkali metals: Computer-aided simulation", *High Temp.* **51**(5), 626–639 (2013).

[46] K. V. Khishchenko, M. V. Zhernokletov, I. V. Lomonosov, and Yu. N. Sutulov, "Dynamic compressibility, release adiabats, and the equation of state of stilbene at high energy densities", *Tech. Phys.* **50**(2), 197–201 (2005).

[47] K. V. Khishchenko, "Equation of state of sodium for modeling of shock-wave processes at high pressures", *Mathematica Montisnigri* **40**, 140–147 (2017).

[48] K. V. Khishchenko, "Equation of state for potassium in shock waves at high pressures", *J. Phys.: Conf. Ser.* **946**, 012082 (2018).

[49] M. H. Rice, "Pressure-volume relations for the alkali metals from shock-wave measurements", *J. Phys. Chem. Solids* **26**(3), 483–492 (1965).

[50] A. A. Bakanova, I. P. Dudoladov, and R. F. Trunin, "Compression of alkali metals by strong shock waves", *Fiz. Tverd. Tela* **7**(6), 1615–1622 (1965).

[51] M. van Thiel (ed.), *Compendium of Shock-Wave Data*, UCRL-50108, Livermore, CA: Lawrence Livermore Laboratory, (1977).

[52] S. P. Marsh (ed.), *LASL Shock Hugoniot Data*, Berkeley, CA: University of California Press, (1980).

[53] A. V. Bushman, M. V. Zhernokletov, I. V. Lomonosov, Yu. N. Sutulov, V. E. Fortov, and K. V. Khishchenko, "Investigation of plexiglas and teflon under double shock loading and in isentropic release waves. Polymers equation of state at high-energy density", *Dokl. Akad. Nauk* **329**(5), 581–584 (1993).

[54] A. V. Bushman, I. V. Lomonosov, V. E. Fortov, K. V. Khishchenko, M. V. Zhernokletov, and Yu. Sutulov, "Shock compressibility and equation of state of a polyimide", *JETP Lett.* **58**(8), 620–624 (1993).

[55] I. V. Lomonosov, A. V. Bushman, V. E. Fortov, and K. V. Khishenko, "Caloric equations of state of structural materials", in: *High-Pressure Science and Technology—1993* ed. by S. C. Schmidt et al., New York: AIP Press, pp. 133–136 (1994).

[56] I. V. Lomonosov, V. E. Fortov, and K. V. Khishchenko, "Model of wide-range equations of polymer materials state under high-energy densities", *Khim. Fiz.* **14**(1), 47–52 (1995).

[57] K. V. Khishchenko, I. V. Lomonosov, and V. E. Fortov, "Equations of state for organic compounds over wide range of densities and pressures", *AIP Conf. Proc.* **370**, 125–128 (1996).

[58] A. V. Bushman, M. V. Zhernokletov, I. V. Lomonosov, Yu. N. Sutulov, V. E. Fortov, and K. V. Khishchenko, "Experimental study of phenylone and polystyrene under the shock loading conditions and isentropic expansion. Plastics state equation under high densities of energy", *Zh. Eksp. Teor. Fiz.* **109**(5), 1662–1670 (1996).

[59] K. V. Khishchenko, "The equation of state for magnesium at high pressures", *Tech. Phys. Lett.* **30**, 829–831 (2004).

[60] K. V. Khishchenko, "Equation of state for tungsten over a wide range of densities and internal energies", *J. Phys.: Conf. Ser.* **653**, 012081 (2015).

[61] K. V. Khishchenko, "Equation of state for titanium at high energy densities", *J. Phys.: Conf. Ser.* **774**, 012001 (2016).

[62] E. Yu. Tonkov, *Phase Diagrams of Elements at High Pressures*, Moscow: Nauka, (1979).

[63] B. Olinger and J. W. Shaner, "Lithium, compression and high-pressure structure", *Science* **219**, 1071–1072 (1983).

[64] E. G. Maksimov, M. V. Magnitskaya, and V. E. Fortov, "Non-simple behavior of simple metals at high pressure", *Phys. Usp.* **48**(8), 761–780 (2005).

[65] V. F. Degtyareva, "Simple metals at high pressures: the Fermi sphere–Brillouin zone interaction model", *Phys. Usp.* **49**(4), 369–388 (2006).

[66] O. Degtyareva, "Crystal structure of simple metals at high pressures", *High Pressure Res.* **30**(3), 343–371 (2010).

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