MOTION CONTROL OF A SPACE ROBOT

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Summary. This article focused on the problem of motion control of a space robot consisting of a body and a telescopic manipulator. The robot is in a state of passive flight. It does not have or does not use propulsions that control movement and orientation of the robot body. Only the actuators installed in the manipulator arm degrees of freedom are used for control of robot's movements. Thus, the movement of the robot is affected only by internal forces. The movement of the manipulator has a noticeable effect on the movement of the robot's body due to the conservation laws of robot's momentum and it's angular momentum relative to the center of mass. It is assumed that the robot's momentum and angular momentum are equal to zero. There are limitations both on the limits of the change in the length of the manipulator arm and the angle of its rotation relative to the body. The problem is solved in a plane formulation. The program motion of the manipulator arm during its movement from the initial position to the final one located in the working is consists of the sequence of the following alternating actions: shortening the manipulator arm length to the minimum value, its rotation relative to the robot body, extending the manipulator arm length to the maximum value, then again its rotation relative to the robot body etc. It is shown that due to these cyclic motions of the manipulator arm relative to the body, the robot body can be rotated to an arbitrary angle. As a result, the working space of a passively flying space robot is significantly larger than the working space of a robot with a fixed body. The working space of the robot in absolute space is a ring bounded y two circles centered at the center of mass of the robot and radii equal to the minimum and maximum distance from the center of mass of the robot to the grasp. Moreover, when constructing the program motion, it is possible to provide not only the output of the robot grasp to a given final position, but also the required (more advantageous for the work) value of the angle between the robot body and the manipulator arm in the final position.

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Key words and Phrases: Space Robot, Motion Control.

1 INTRODUCTION

Space robots, consisting of the main body (body) and equipped with one or more manipulators, are promising for carrying out various types of work in outer space and in the orbit of the Earth satellite for repair, maintenance and construction of various objects (space stations, orbital telescopes, etc.), as well as for the work on the removal of space debris. A review of the work in this direction is given in [1, 2].

In this work we consider motion of a passively flying space robot. The robot consists of a body and manipulator arm. It does not have or does not use propulsions that control movement and orientation of the robot body. The movement of the manipulator has a significant influence on the movement of the robot body due to the conservation laws of robot's momentum and angular momentum [1–7].

In [1–4] considered the plane problem of the motion control of passively flying robot consisting of the body and the manipulator. The manipulator has kinematic redundancy and consists of at least three hinged unit links. There are no constraints on the limits of variation of the deviation angles of the manipulator arm. It is assumed that the robot's momentum and angular momentum are equal to zero. Authors of articles [1, 2] and [3, 4] use different methods for motion control of the robot. But, despite the efforts of the authors, they obtained that it is impossible to move the robot's gripper in a significant portion of kinematically achievable final positions located in the robot's workspace.

In [5–7], it is shown that this result appears to be erroneous and caused by the methods used to the motion control of the robot based on the local principle of their formation. It takes transition to a global principle of constructing programmed motion of the robot, which would require the performance of special, not obvious movements in advance for the realization of this goal. On a simple model example, of planar motion of the robot with telescopic manipulator analytically shown that it is possible to provide the motion of the gripper of the robot from an arbitrary initial to an arbitrary kinematically achievable final position. The problem was solved under the assumption that there are constraints on the range of varying of the length of manipulator and there are no constraints on the range of varying of the angle of manipulator arm rotation relative to its body.

In this work, which is the evolution of [5–7], this problem is solved if there are constraints as on varying the length of the manipulator, and so on the value of the angle of manipulator rotation about the body. It is shown that the workspace of the robot significantly more than that of a similar robot with a fixed body. By special motions of the manipulator can ensure a reversal of space robot and move the gripper of the robot from an arbitrary initial to an arbitrary final position, if they are located inside the ring with the center in the robot's center of masses is bounded by circles with radii equal to the minimum and maximum distance from the gripper to the robot's center of masses. Furthermore, it is shown that it is possible to provide the required (the most convenient for the execution of works) a value of the angle between the robot body and manipulator arm in the final position.

2 THE PROBLEM STATEMENT

Consider planar motion of a passively flying space robot. Fig. 1 shows a construction scheme of the robot, which consists of the body with mass m_1 and the telescopic manipulator attached to the body at its center of mass B. The moment of inertia of the body about its center of mass equal J_1. The manipulator consists of a cylinder that rotates relative to the point of his suspension to the body, and the rod is moved along the telescopic link of the arm. We denote m_2, m_3, J_2, J_3 respectively the mass of the cylinder and the rod of the manipulator and their moments of inertia about their centers of mass C_2 and C_3. When the robot moves only internal forces act in the degrees of freedom of the manipulator. There are a conservation laws of the robot's momentum and its angular momentum about the center of mass. Consider the problem of motion control of the robot at zero values of the momentum and its angular momentum. The center of mass of the robot C remains stationary and its position is taken as the beginning of inertial (orbital) coordinate system $C\eta\zeta$.

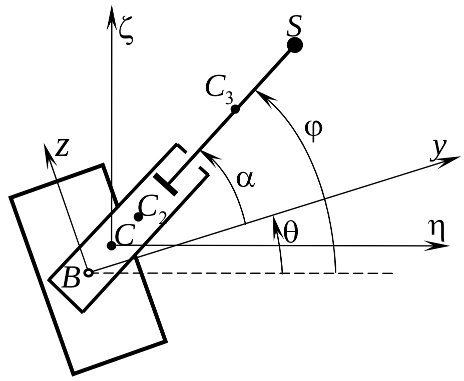

Figure 1. The constructive scheme of the space robot

The coordinate system Byz associated with the robot body, the axis Bz is constructive vertical and the axis By – the longitudinal axis of the body. The position of the robot in the absolute coordinate system $C\eta\zeta$ is defined by the coordinates of the body center of mass, the angle of body rotation θ, the angle between the axis By and manipulator arm α and the length of the arm $l = BS$. Point S – gripper of the manipulator. The angle φ specifies the orientation of the manipulator arm in absolute space:

$$\varphi = \theta + \alpha \quad . \tag{1}$$

The center of mass of the cylinder C_2 lies on the axis of the telescopic link at the distance $r_2 = BC_2$ from the point of its suspension to the body. The center of mass of the rod C_3 lies on the axis of the telescopic link at the distance $r_3 = SC_3$ from the gripper S. The center of mass of the robot C lies on the axis of telescopic link at the distance $\rho_B = BC$. The length of the manipulator l and the distance from the center of mass of the robot to the gripper $\rho = l - \rho_B$ are related by [5, 6]

$$(m_1 + m_2 + m_3)\rho = (m_1 + m_2)l - m_2 r_2 + m_3 r_3 \quad . \tag{2}$$

The angular momentum conservation law about the center of mass has the form [5, 6]

$$k\dot{\varphi} + J_1\dot{\theta} = k\dot{\alpha} + (k + J_1)\dot{\theta} = 0 \quad , \tag{3}$$

where

$$k = k(l) = \mu_1(l - \lambda)^2 + \mu_2 \quad , \quad \mu_1 = \frac{(m_1 + m_2)m_3}{m_1 + m_2 + m_3} \quad , \quad \lambda = \frac{m_2 r_2}{m_1 + m_2} \quad ,$$
$$\mu_2 = J_2 + J_3 + \frac{m_3^2 r_3^2}{m_1 + m_2 + m_3} + \frac{m_1 m_2 r_1^2}{m_1 + m_2} \quad . \tag{4}$$

Suppose that there are constraints on the limits of the length of the manipulator and theangle of its rotation relatively the body

$$l \in [\, l_{\min} \;, l_{\max} \,] \quad , \quad \alpha \in [\alpha_{\min}, \alpha_{\max}] \quad . \tag{5}$$

The position of the gripper of the robot is determined by the polar coordinates $\rho = CS$ and φ. The purpose of control is to move the robot's gripper from any arbitrary start position (ρ_0, φ_0) to arbitrary final position (ρ_D, φ_D) inside the workspace. The workspace is limited by two circles with center in the center of mass of the robot C and the radii (2) $\rho_{\min} = \rho(l_{\min})$ and $\rho_{\max} = \rho(l_{\max})$. In addition, we require that the angle between the robot body and its arm in final position was a predetermined value α_D (for example, the most convenient to perform the work). From equations (1–2) it follows that for this is necessary and sufficient to translate the robot from initial state θ_0, α_0, l_0 to final state θ_D, α_D, l_D – where $\theta_D = \varphi_D - \alpha_D$, $l_0 = l(\rho_0)$, $l_D = l(\rho_D)$.

3 MOTION PLANNING

The robot motion will be implemented in such a way that it was divided into time intervals (stages), during each of which operates only one actuator of manipulator. Either changes the length of the manipulator under a fixed angle α, in this case the angle θ remains constant (3), or change the angle of arm rotation relative to the body at a fixed length of the manipulator, then (3)

$$\dot{\theta} = -b(l)\dot{\alpha} \quad , \tag{6}$$

where

$$b(l) = \frac{k(l)}{k(l) + J_1} \quad .$$

Proposition 1. The ratio $b(l)$ is an increasing function.

By (4) $k(l) > 0$ is a monotonically increasing function of l in the domain (5), because $l_{\min} > r_2 > \lambda$. If $l_2 > l_1$, then $b(l_2) - b(l_1) = \dfrac{J_1\big(k(l_2) - k(l_1)\big)}{\big(k(l_1) + J_1\big)\big(k(l_2) + J_1\big)} > 0$.

Integrating (6) for $l = \text{const}$ from the initial position $\tilde{\alpha}, \tilde{\theta}$, obtained

$$\theta = f(\tilde{\theta}, l, \alpha, \tilde{\alpha}) = \tilde{\theta} - b(l)(\alpha - \tilde{\alpha}) \quad , \tag{7}$$

then a turn of the manipulator arm on the angle $\Delta\alpha = \alpha - \tilde{\alpha}$ relative of the robot body leads to its rotation at the angle $\Delta\theta = \theta - \tilde{\theta}$. On the plane of variables α and θ this dependence $\theta(\alpha)$ has the form of the family of straight lines, the slope of which depends at l. Let's call the shaded domain and together with its boundaries in Fig. 2 the attainability domain from the position $\tilde{\alpha}, \tilde{\theta}$ at the fixed $l \in [l_{\min}, l_{\max}]$. Note that at each stage the robot motion its final position does not depend on the law of variation of the manipulator coordinates l and α, but determine only by the initial and final value of this coordinate.

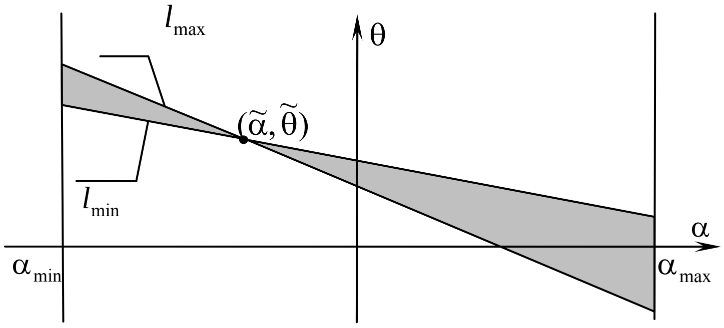

Figure 2. The attainability domain of the position $\tilde{\alpha}, \tilde{\theta}$

Proposition 2. The robot can move from start position α_0, θ_0, l_0 to terminal position α_D, θ_D, l_D, if they satisfy the constraints (5) and represents the point α_D, θ_D lies in the attainability domain of point α_0, θ_0.

In this case, from (4, 7) it follows that

$$\theta_D = \theta_0 - b(l^*)(\alpha_D - \alpha_0) \quad , \tag{8}$$

where l^* is determind from equations (4), (8)

$$l^* = \lambda + \sqrt{-\frac{1}{\mu_1}\left(\mu_2 + \frac{J_1(\theta_D - \theta_0)}{\theta_D - \theta_0 + \alpha_D - \alpha_0}\right)} \quad ,$$

and in accordance with the proposition 1 $l^* \in [l_{\min}, l_{\max}]$.

Proposition 3. The robot can move from any arbitrary start position α_0, θ_0, l_0 to arbitrary final position α_D, θ_D, l_D if they satisfy the constraints (5).

If the final position of the imaging point α_D, θ_D lies inside the attainability domain from its initial position, the programmed motion is constructed in accordance with the proposition 2.

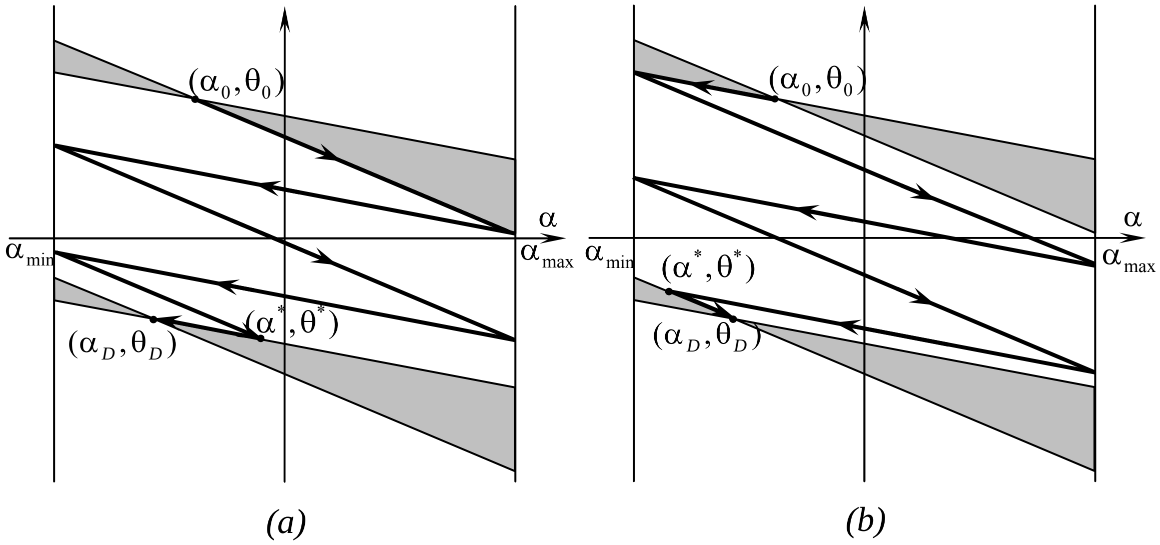

Figure 3. The trajectory of the imaging point for the case when its final position lies below the attainability domain from the initial position

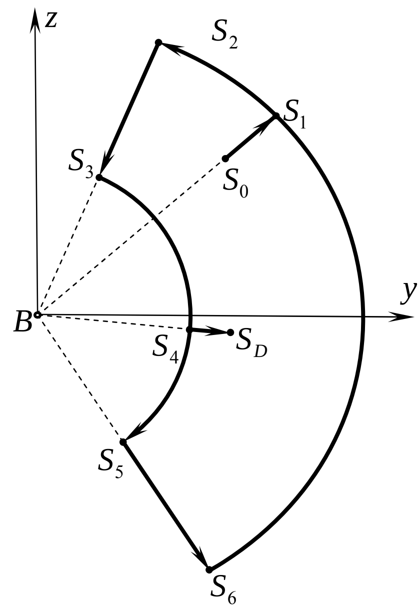

Figure 4. The trajectory of the manipulator arm gripper in relative coordinate system connected with the robot body.

If the final point α_D, θ_D lies below the attainability domain from the initial position α_0, θ_0, there are two ways of planning a programmed motion. In Fig. 3 shows the corresponding trajectory of the imaging point on the plane α, θ.

When using the first method (Fig. 3,a) the gripper of the manipulator in the relative coordinate system Byz connected with the robot body moves along the trajectory shown in Fig. 4. The initial position of the gripper is indicated S_0 , and the final – S_D. If the final position of the imaging point α_D, θ_D lies near the attainability domains from the initial position α_0, θ_0, the gripper of the manipulator moves along the path $S_0 S_1 S_2 S_3 S_4 S_D$.

Otherwise, the gripper of the manipulator moves along a trajectory $S_0 S_1 \_ S_2 S_3 S_5 S_6 S_2 \_ S_3 S_4 S_D$, and allocated closed trajectory of the gripper $S_2 S_3 S_5 S_6 S_2$ can be repeated several times. In this cyclic motion of the gripper in the sections $S_2 S_3$ and $S_5 S_6$ change only the length of the arm and the angle of rotation of the robot body in absolute space remains is unchanged. In the section $S_3 S_5$ of the manipulator rotated relative to the body clockwise under $l = l_{\min}$ and in accordance with (7), the robot body is rotated counterclockwise by the angle modulo equal to $|\Delta\theta_1| = b(l_{\min})(\alpha_{\max} - \alpha_{\min})$. In the section $S_6 S_2$ the manipulator arm rotated relative to the body counterclockwise under $l = l_{\max}$ and in accordance with (7), the robot body is rotated clockwise by the angle is modulo equal to $|\Delta\theta_2| = b(l_{\max})(\alpha_{\max} - \alpha_{\min})$. In view of proposition 1 $b(l)$ is a monotonically increasing function of the manipulator arm length l, and, consequently, $|\Delta\theta_2| - |\Delta\theta_1| = \left(b(l_{\max}) - b(l_{\min})\right)(\alpha_{\max} - \alpha_{\min}) > 0$. As a result of each cycle of motion of the gripper of manipulator arm on closed section $S_2 S_3 S_5 S_6 S_2$ robot body turned clockwise at the angle $|\Delta\theta_2| - |\Delta\theta_1| > 0$.

If the final position of the imaging point α_D, θ_D lies above the attainability domain from the initial position α_0, θ_0, the programmed motion is planning analogous.

12 CONCLUSIONS

The planar motion of space robot in a state of passive flight under the assumption that the momentum and angular momentum of the robot equal to zero is investigated. Considered robot consist of a body and telescopic manipulator. It assumed that there are the constraints as to the limits of the length of the manipulator arm, and so at the its rotation angle about the body. It is shown that due to special cyclic motions of the manipulator relative to the body it is possible to turn the robot body to an arbitrary angle. In the result, the workspace of a passively flying space robot (the possible final position of the robot gripper) is more larger than the workspace of the robot with a fixed body. The workspace of the robot in absolute space is a ring bounded by two circles centered with the center of mass of the robot and the radii equal to the minimum and maximum distance from the center of mass of the robot to gripper. Moreover, in programmed motion planning can be provided not only the release of the gripper of the robot to a given final position, but also the required (convenient for work) the value of the angle between the robot body and manipulator arm in the end position. It should be expected that a similar result can be obtained for more complex robot design schemes in which the center of mass of the body does not coincide with the point of attachment of the manipulator arm to the body and (or) the arm of the manipulator consists of pivotally connected links (possibly with kinematic redundancy). For this purpose it is necessary to use a numerical solution of the boundary problem of moving the robot grasp from a given initial to a specified final position within the working area.

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