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DSMC MODELING OF ELECTRON HEATING FOR THE NONLINEAR COLLISION KINETIC EQUATION WITH QUASILINEAR DFFUSION

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Summary. Solution of the time dependent spatially uniform nonlinear collision kinetic equation in the course of heating is studied numerically. Coulomb collisions are treated with the Landau-Fokker-Planck collision integral. The considered heating source is a quasilinear diffusion operator with variable coefficient acting in a full velocity space. DSMC method is used to investigate the process. The time-dependent solutions are examined.

1 INTRODUCTION

In many important cases one should treat plasma transport kinetically. Examples are: the electron heat transport in inertial confinement fusion (nonlocal transport); propagation of the heat bursts, caused by edge localized mode (ELM), into scrape-off layer (SOL) of tokamak. In more general sense the solutions of the Landau - Fokker-Planck (LFP) (LFP) equation[1-3], which is one of the key ingredient of plasma kinetic equation, have much broader interest ranging from plasma physics to stellar dynamics (e.g. see Refs.[4] - [8] and the references therein).

The spatially homogeneous kinetic equation for Coulomb interaction was first published by L.D. Landau in 1936. The collision operator for charged particles under assumption of grazing collisions was obtained by Landau as an approximation of the Boltzmann integral when the mean energy of Coulomb interaction is small versus the mean kinetic energy, i.e. en. The Landau equation is often in literature called the Landau-Fokker-Planck equation after its rediscovery in the Fokker-Planck form in the important paper[2]. Beginning with the related publication[3], a lot of work is done on numerical methods for the Landau equations based on finite difference schemes. We indicate the review[4] on the subject which contains many references.

In practice, kinetic solution of plasma transport problems can be done only numerically which is very difficult and time consuming. In addition, complex nonlinear kinetic codes require careful benchmarking, which is not a trivial problem on its own. However, in many cases it worthwhile to analyze simpler models, solution of which, nevertheless, exhibit some important features of the problem of interest. We notice that such models also help to benchmark complex kinetic codes. For example, the transport of electrons along the magnetic field lines from hot upstream region of the SOL to cold diverter region during ELM burst, resulting in the enhancement of the tail and anisotropization of electron distribution function in diverter, can be mimicked by a proper heating term in a much simpler time-dependent kinetic equation[9]. The time dependent solutions in self-similar variables for isotropic

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nonlinear kinetic equation are examined analytically and numerically in[10].

In the above cited papers the LFP equation is treated numerically with the deterministic finite difference approach. Being rather precise for the detailed description of the function distribution this approach yields to the DSMC methods in problems with high dimensionality. Besides for solving the Vlasov kinetic equation (for collisionless plasmas) the particle methods are usually employed[11], which are in particular matched with DSMC methods.

This paper presents numerical results of the DSMC treatment of the space uniform nonlinear electron kinetic equation with heating terms which is a diffusion operator with a variable quasilinear diffusion coefficient. The qiasilinear diffusion operator is a usual well-known way to describe a wave-particle interaction and the wave power absorption mechanism due to Landau damping. The main goal of this work is to suggest an algorithm for joint action of LFP and diffusion operators and to show the possibilities of a new DSMC method[12,13].

First we give the statement of the problem, shortly describe the idea of the DSMC method for the LFP equation and its algorithm. Then an algorithm for the diffusion operator is suggested. The interplay between the effects of Coulomb collisions and the diffusion operator is studied. Comparison with the self-similar solutions[10] obtained in the course of quasilinear heating is provided.

2 PRELIMINARES

We consider an arbitrary spatially homogeneous mixture of rarefied gases. Let $\{f_i(v,t), i=1, ..., n\}$ be distribution functions of particles with masses $\{m_i,\ i=1, ..., n\}$, respectively. The independent variables $v \in \mathbf{R}^3$ and t $\geq$ 0 stand for velocity and time, respectively. Spatial densities $\{\rho_i(\mathrm{t}), \mathrm{i}=1, ..., \mathrm{n}\}$ are given by integrals

$$\rho_\alpha = \int_{\mathbf{R}^3} d\boldsymbol{v}\ f_\alpha(\boldsymbol{v},t). \tag{1}$$

The system of Boltzmann kinetic equations for $f_i(v,t)$ reads

$$\frac{\partial f_i}{\partial t} = \sum_{j=1}^{n} Q_{ij}(f_i, f_j), \quad i,j=1,...,n, \tag{2}$$

where

$$Q_{ij} = \int_{\mathbb{R}^3\times S^2} g_{ij}\left(|\boldsymbol{u}|, \frac{\boldsymbol{u}\cdot\boldsymbol{\omega}}{|\boldsymbol{u}|}\right)\left[f_i(\boldsymbol{v}')f_j(\boldsymbol{w}') - f_i(\boldsymbol{v})f_j(\boldsymbol{w})\right]d\boldsymbol{\omega}\, d\boldsymbol{w}, \quad |\boldsymbol{u}| = |\boldsymbol{v}-\boldsymbol{w}|,$$

$$\boldsymbol{v}' = \frac{1}{m_i+m_j}\left(m_i\boldsymbol{v} + m_j\boldsymbol{w} + m_j|\boldsymbol{u}|\boldsymbol{\omega}\right), \quad \boldsymbol{w}' = \frac{1}{m_i+m_j}\left(m_i\boldsymbol{v} + m_j\boldsymbol{w} - m_j|\boldsymbol{u}|\boldsymbol{\omega}\right) \tag{2}$$

Functions $g_{ij}(u,\mu)$ are expressed by formulas

$$g_{ij}\left(|\boldsymbol{u}|,\mu\right)=g_{ji}\left(|\boldsymbol{u}|,\mu\right)=|\boldsymbol{u}|\,\sigma_{ij}. \tag{3}$$

where $\sigma_{ij}\left(u,\ \mu\right)$ is the differential cross section (in the center of mass system of colliding particles of type i and j) of scattering at the angle $\theta\ =\ \arccos\left(\mu\right),\ \ |\mu|\leq\ 1.$

The system of Boltzmann equations (BE) (2) is interesting for us merely as a starting point to pass to the Landau equations. For such transition one needs to choose a special kind of functions $g_{ij}\left(u,\mu\right)$ (this choice is based on the result obtained in [14]). In the next section we give an idea of how to implement the Boltzmann integral for DSMC modeling of Coulomb collisions and, consequently, the LFP integral.

The system of LFP equations for space uniform plasmas with a quasilinear operator reads

$$\frac{\partial f_k}{\partial t}=\sum_{j=e,i}^{n}Q_{kj}^{L}(f_k,f_j)\ +\frac{\partial}{\partial\boldsymbol{v}}D\left(\boldsymbol{v},t\right)\frac{\partial f_k}{\partial\boldsymbol{v}}\ ,\quad k,j\ =\ e,\ i, \tag{4}$$

where $Q_{kj}^{L}(f_k,f_j)$ - Landau collisional integrals

$$Q_{kj}^{L}(f_k,f_j)\ =\ 2\pi L\quad\sum_{j=e,i}^{n}\ \frac{e_k^2e_j^2}{m_k^2}\frac{\partial}{\partial v^\alpha}\int_{\mathbb{R}^3}dw\ R^{\alpha\beta}\left(u\right)\left(\frac{\partial}{\partial v^\beta}-\frac{m_k}{m_j}\frac{\partial}{\partial w^\beta}\right)f_k(v)f_j(w) \tag{5}$$

with the symmetric kernel

$$R^{\alpha\beta}\left(u\right)=\frac{u^2\delta^{\alpha\beta}-u^\alpha u^\beta}{u^3}\ ,\ u\ =\ v\ -\ w;\quad\alpha,\ \beta\ =1,\ 2,\ 3. \tag{6}$$

For solving the kinetic equation for plasmas it is natural to use standard splitting methods, i.e. to consider separately (a) continuous motion of electrons and ions in external fields (the quasilinear diffusion in our case), and (b) Coulomb collisions. The splitting procedure is formally quite similar to what we do in simulation of neutral gases by Monte Carlo methods[15] We shall firstly consider the second stage (b), related to Coulomb collisions, and show. the idea of the DSMC method for one sort of particles.

3 DSMC METHOD FOR THE LANDAU-FOKKER-PLANCK EQUATION

For the sake of clarity we use one sort of particles. Let us consider the BE (2)

$$f_t\ =\ \hat{J}\left[f,f\right]=\int_{\mathbf{R}^3\times\mathbf{S}^2}dvdw\ g\left(|u|,\mu\right)\left[f\left(v',t\right)f\left(w',t\right)-f\left(v,t\right)f\left(w,t\right)\right]\ ,$$

$g\left(|u|,\mu\right)=u\sigma\left(u,\ \mu\right)$. Assume that the scattering angle θ satisfies the condition (grazing

collisions) $0 \le \theta \le \delta$, $0 < \delta << 1$. In other words $g(u,\mu) \equiv 0$ if $-1 < \mu < 1 - c\delta^2$. Then we formally obtain the Landau equation

$$\frac{\partial f}{\partial t} = \hat{I}[f,f] = \frac{1}{8}\frac{\partial}{\partial v_i}\int_{\mathbf{R}^3} dw\, g_{tr}(|u|)\left(u^2\delta_{ij} - u_i u_j\right)\left(\frac{\partial}{\partial v_i} - \frac{\partial}{\partial w_j}\right) f(v) f(w) \qquad (7)$$

with

$$g_{tr}(|u|) = \frac{1}{2}\int d\mu\, d\phi\, g(u,\mu)\,(1-\mu) = \frac{1}{2}u\int d\mu d\phi\, \sigma(u,\mu)\,(1-\mu) = u\,\sigma(u)_{tr}.$$

Hence, the Landau equation can be solved by using the Boltzmann equation with appropriate $g(u,\mu)$.

Key idea is to choose $g(u,\mu)$ in such a way that the Monte-Carlo solution of the Boltzmann equation can be constructed in the simplest way. We choose

$$g(u,\mu;\varepsilon) = \frac{1}{2\pi\varepsilon}\delta\left[1-\mu-\varepsilon u^{\alpha}\right]$$

where ε is a small parameter,

$$1-\varepsilon u^{\alpha} = \begin{cases} 1-\varepsilon u^{\alpha}, & \varepsilon u^{\alpha} \le 2, \\ +1, & \text{otherwise.} \end{cases}$$

The function $g(u,\mu;\varepsilon)$ means that the scattering always occurs at fixed angle $\sim \arccos\left[1-\varepsilon u^{\alpha}\right]$ for collision of particles and that $\mu = \pm 1$ means 'no scattering'. This scattering law is convenient for the application of the Monte Carlo method. Another advantage of this approximation is that the total collision frequency is constant:

$$g_{ij}^{tot}(u,\varepsilon) = 2\pi\int_{-1}^{1} d\mu\, g_{ij}(u,\mu;\varepsilon) = \frac{1}{\varepsilon}.$$

Such an approximation can be called quasi-Maxwellian, since the total collision frequency (for any pair of sorts i and j, including the case $i = j$) is independent of velocities. Note that ε has dimensionality $[t][l]^{-3}$, we ignore this fact considering ε simply as a small parameter (note that $\mu = \pm 1$ means 'no scattering'). Let

$$J_+[f,f] = \int dwd\omega \frac{\delta\left[1-\mu-\varepsilon u^\alpha\right]}{2\pi} f(v',t) f(w',t) \quad .$$

Then BE reads

$$f_t = \frac{1}{\varepsilon} J_+[f,f] - \frac{1}{\varepsilon} f, \tag{8}$$

provided

$$\int_{R^3} dv \; f(v,t) = 1.$$

In the limit $\varepsilon \to 0$ we obtain (8), where

$$g_{tr}(u) \equiv \frac{1}{\varepsilon} \int_{-1}^{1} d\mu \delta[1-\varepsilon u^\alpha - \mu](1-\mu) = u^\alpha \quad if \quad \varepsilon u^\alpha \le 2$$

(otherwise $g_{tr}(u) = 0$ - this does not matter in the limit $\varepsilon = 0$). Hence, by solving BE with 'very small' $\varepsilon > 0$ we actually solve the following Landau equation

$$\frac{\partial f}{\partial t} = \hat{I}[f,f] = \frac{1}{8} \frac{\partial}{\partial v_i} \int_{R^3} dw \, u^\alpha \left(u^2 \delta_{ij} - u_i u_j\right) \left(\frac{\partial}{\partial v_i} - \frac{\partial}{\partial w_j} \right) f(v) f(w) .$$

Thereby the Monte Carlo scheme is made up of the following steps. We have BE (9)

$$\varepsilon \frac{\partial f}{\partial t} + f = J_+[f,f] \; .$$

The quasi Maxwellian approximation

$$\frac{\partial f}{\partial t} \approx \frac{1}{\varepsilon} \left[f(t+\varepsilon) - f(t) \right]$$

leads to the following equality

$$\varepsilon \frac{f(t+\varepsilon) - f(t)}{\varepsilon} + f(t) = f(t+\varepsilon) = J_+[f,f].$$

Hence, the distribution function on the subsequent time step can be defined explicitly from the gain term of the collisional operator $f(t+\varepsilon) = J_+[f,f]$:

$$f(t+\varepsilon) = J_+[f,f] \;=\; \frac{1}{2\pi}\int\limits_{R^3\times S^2} dw\, d\omega\, \delta\left[1-\varepsilon u^{\alpha}-\frac{u\cdot\omega}{u}\right] f(v',t) f(w',t)\,,$$

where $u = v - w$, $v' = U + u\omega/2$, $w' = U - u\omega/2$, $U = (v + w)/2$. Thus we have an explicit scheme. Our method is the usual simulation of this relation with unit vector ω given in the special coordinate system by two angles such that

$$(\theta,\ \varphi) \propto \begin{cases} \cos\theta \;=\; \mu = 1 - \varepsilon u^2 & \text{and} \\ \varphi \ \text{ is uniformly distributed on } & [0, 2\pi]. \end{cases}$$

Let ε be a small positive number, and

$$\tau \;=\; \varepsilon u\, \sigma_{tr}(u), \qquad \mu(\tau) \;= \begin{cases} 1 - 2\tau, & 0 \le \tau \le 1 \quad \text{and} \\ -1, & \text{otherwise.} \end{cases}$$

The cross section of charged particles is given by the Rutherford formula

$$\sigma(u,\ \theta) \;=\; const \cdot u^{-4} \sin^{-4}\frac{\theta}{2}.$$

With angular cut-off at small angle θ_{min}, we can approximate the momentum transfer cross section by

$$\sigma_{tr}(u) \approx const \cdot u^{-4} \int\limits_{\theta_{\min}}^{\pi} d\theta\, \frac{\cos\theta/2}{\sin\theta/2} = const \cdot u^4\,.$$

Therefore, we have $\tau \sim \varepsilon u^{-3}$ in this case.

As a result the *simulation algorithm* can be summarized like that.

• For N particles a set of N velocity vectors $V(0) = v_1(0) \le v_2(0) \le \ldots \le v_N(0)$ is generated.

• Repeat the following steps:

- Advance time $t \to t + \Delta t$, $\Delta t = 2\varepsilon / N$.

- Choose any pair $i \ne j$, $j = 1,\ldots,N$ randomly.

- Compute the center of mass velocity $\boldsymbol{U}=(\boldsymbol{v}_i + \boldsymbol{v}_j)/2$ and relative velocity $\boldsymbol{u} = \boldsymbol{v}_i - \boldsymbol{v}_j$.

- Find $\theta = \arccos\mu(\tau)$ and pick φ uniformly distributed in $[0,\ 2\pi]$. Then $\omega = (\theta,\ \varphi) \in S^2$ is the new relative velocity $\boldsymbol{u}' = u\,\boldsymbol{\omega}$

- Compute the new velocities $\boldsymbol{v}_i = \boldsymbol{U} + \boldsymbol{u}'/2$ and $\boldsymbol{v}_j = \boldsymbol{U} - \boldsymbol{u}'/2$ and obtain N new velocity vectors $V(\varepsilon) = v_1(\varepsilon),\ldots,v_N(\varepsilon)$.

It is clear that the algorithm satisfies all physical conservation laws. Note that any another model cross section can be chosen for the approximation of the LFP equation.

4 STOCHASTIC MODELING OF THE QUASILINEAR DIFFUSION

Below we consider an interaction of RF waves with a plasma that is described by a LFP equation with an added quasilinear term

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \boldsymbol{v}} D(v,t) \frac{\partial f(\boldsymbol{v},t)}{\partial \boldsymbol{v}} + Q_{ee}^{L}(f,f), \qquad v = \sqrt{v_x^2 + v_y^2 + v_z^2}\ . \tag{10}$$

At the beginning we consider the first stage (a) - the action of the quasilinear diffusion operator and describe the algorithm of its stochastic modeling for only one direction

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v_z} D(\sqrt{v_z^2}, t) \frac{\partial f(v,t)}{\partial v_z} \tag{11}$$

(for the directions x, y and for the quasilinear coefficient $D(v)$ the algorithm will be the same).

In order to apply the Monte Carlo method to the diffusion process we use in some way the intuitive approach based on the integro-interpolation method for the discretization of the diffusion equation. In this case the starting problem is the original analytical statement - the Laplace differential equation. Then the simplest random process can be defined following the assumption of discreteness of the diffusion unit event - the Brownian motion (the Fokker-Planck equation). Diffusion equation (11) can be solved in the following way.

Let us choose the small parameter ε corresponding to the time step Δt and a large number N of particles (velocities) that are distributed randomly as consistent with the initial condition, $f|_{t=0} = f_0(v)$, $\int_{-\infty}^{\infty} f_0(v)\, dv = 1$ (one can say about one particle but many iterations).

• Let particles have coordinates $V(0) = v_1(0) \le v_2(0) \le \ldots \le v_N(0)$.

Repeat the following steps:

• The random number $\eta \in [0,\ 1]$ is picked out for each of particles.

• After this the particle is shifted to the right by

$$\Delta v = \sqrt{2D\varepsilon} + D_z'\ \varepsilon\ ,$$

if $\eta \ge 1/2$, or to the left by

$$\Delta v = \sqrt{2D\varepsilon} - D_z'\varepsilon,$$

if $\eta < 1/2$, or vice-versa.

• New locations of particles are denoted as (the numeration order does not important)

$V(\varepsilon) = v_1(\varepsilon),\ v_2(\varepsilon),\ \ldots,\ v_N(\varepsilon)$.

We remind that $D(v_z, t) \ge 0$. The process is repeated to obtain the subsequent values of $V(\varepsilon)$, and so on.

Then the limit $\varepsilon \to 0$ solves the diffusion equation (11).

Really, let $f(v,\ t)$ is a probability of particle to be located at the point v at the time moment t. Our scheme corresponds to the equation

$$f(v,\ t+\varepsilon)\ = \frac{1}{2}\left\{ f\left(v + (\sqrt{2D\varepsilon} + D_z^{'}\varepsilon, t)\ \right) + f\left(v - (\sqrt{2D\varepsilon} - D_z^{'}\varepsilon\cdot, t)\ \right)\right\}.$$

If $f(v,\ t)$ is a "good" function then at $\varepsilon\ \to\ 0$ we have

$$f(v,\ t)\ +\ \varepsilon f_t(v,\ t)\ +\ O(\varepsilon^2) = \frac{1}{2}\ [2f(v,\ t)\ +\ (\sqrt{2D\varepsilon})^2\, f_{zz}(v,\ t)\ + 2D_z^{'}\, f_z^{'}\varepsilon\ +\ O(\varepsilon^2)].$$

From this we obtain $f_t(v,\ t) = Df_{zz}(v,\ t)\ +\ D_z^{'}\cdot f_z^{'}\ + O(\sqrt{\varepsilon}),$ what means an approximation of the $O(\sqrt{\Delta t})$ order. More rigorously, $f(v,\ t)$ should have bounded time derivatives up to the second order and directional derivative v up to the third order (but for physics it does not matter). Obviously, the same procedure is valid for the qusilinear diffusion coefficient acting in all directions.

5 EFFECT OF THE LFP OPERATOR AND THE DIFFUSION OPERATOR COMBINED ACTION

Here we consider the case when coefficient of quasilinear diffusion $D(v,\ t)$ increases with velocity increasing. In [10] a special class of functions, for which it is possible to construct a self-similar solution is considered. We tested our simulation with these solutions.

To solve (10) dimensionless variables are used in the following units: electron thermal velocity $v_{th}\ = \sqrt{2T_{e/}\ /\ m_e}$, electron-electron collision time $t_e\ = f_e 4\pi\ \Gamma$ and distribution function $\sim\ \rho / 4\pi v_{th}^3$. Here $\Gamma\ =\ 2\pi e^4\ L / m_e^2$, where L is the Coulomb logarithm. Also an energetic variable $\xi\ =\ mv^2\ /\ 2T\ =\ v^2\ /\ v_{th}^2$ is used. The particle density ρ , that is constant in time, and temperature T (average energy) are defined through integrals

$$\rho\ = \int_0^\infty dv\ v^2\ f(v,t) = 1, \qquad T = \frac{1}{3\rho}\int_0^\infty dv\ v^4\ f(v,t),$$

in spherical coordinates. Equilibrium Maxwellian distribution takes a form

$$f_M(v,t) = \frac{4}{\sqrt{\pi}}\ v_{th}^3(t)\ \exp\left[-\frac{v^2}{v_{th}^2(t)}\right],$$

which in case of heating depends on time.

The only case, when the equation (10) formally admits stationary solutions, $\partial f\ /\ \partial t\ = 0$, corresponds to $D(\xi,\ t)\ =\ D(\xi)\cdot D(t)\ \approx\ D(\xi)\ /\ \sqrt{T(t)}$. Taking $D(\xi)\xi^p\ =\ D_0\xi^p$, where

D_0 is the normalization constant and $1 > p > 0$ is an adjustable parameter, we find the distribution function[10]

$$f(\xi \to \infty) \propto \exp\left\{-const\,\frac{\xi^{1-p}}{1-p}\right\}, \qquad 1 > p > 0. \tag{12}$$

For $p = 0$ we have the Maxwellian distribution which is also the solution to the LFP operator. For $p = 1/2$ we obtain the enhanced tail $f(\xi \to \infty) \propto exp\{-\sqrt{\xi}\}$. Consequently, to verify our computational method two variants are considered (in Cartesian coordinates): $D(v) \sim v^p$, $p = 0, 1$. For $p = 0$ the solution of the diffusion equation is the Maxwell distribution. Then the solution of the Cauchy problem for (10) with $p = 0$ is the Maxwellian with variable temperature.

We compare our DSMC results with the usage of numerically computed moments

$$M_\alpha^{2n}(t) = \frac{1}{N}\sum_{j=1}^{N_\alpha}\left|\boldsymbol{v}_j\right|^{2n}, \tag{13}$$

together with the analytical expressions

$$m_\alpha^{2n}(t) = \int_{\Re^3} |\vec{v}|^{2n} f_\alpha(\vec{v},t)d\vec{v}. \tag{14}$$

Note that $n = 1$ corresponds to the second distribution function moment which is the averaged energy (temperature).

In the course of heating, the coefficient of the qiasilinear diffusion is taken in a form $D(v, t) = 1/\sqrt{T}$ for $p = 0$ and $D(v, t) = v/T$ for $p = 1$. This choice provides equal growth of temperature in time for both cases $p = 0, p = 1$ and we can compare higher moments that describe the behavior of the distribution function tails

$$\frac{1}{N}\sum v_i^4, \quad \frac{1}{N}\sum v_i^6, \quad \frac{1}{N}\sum v_i^8, \ldots$$

The initial distribution is taken as $f_0(v) = \delta(v-1)/v^2$. For DSMC method it means that the initial velocities are uniformly distributed on the unit sphere, i.e. $|v_i| = 1$. Note, initially all distribution function moments are equal to unit.

Fig.1 shows comparison of temperature (the second moment $M_2(t)$) for two diffusion coefficients: $p = 0$ and $p = 0.5$. The same balance in temperature variation in time preserves for all considered cases. For the case $p = 0$ we have to obtain the Maxwellian distribution function. Distribution moments obtained numerically with those obtained for Maxwellian distribution are compared in Fig.2. After some while ($t > 2.5$) we have good qualitative and quantitative agreement.

For $p = 1$ comparison with the Maxwellian moments $p = 0$ show a substantial enhancement of the distribution tails - Fig.3. The lower moments are closer to each other. It

means that in the thermal velocity region functions for different p are close to Maxwellian. For better presentation moments are normalized on the second moment 10 (energy). Distribution functions for different time moments are shown in Figs.4, 5. Fig.6 shows comparison of normalized moments $M_{6,10}$ and $m_{6,10}$. As can be clearly seen the formation of the distribution function tails occur later than electron collusion time (t = 1). One can see that the high order moments of the distribution are accompanied with stochastic oscillations even for a big number of N. Finally, Fig. 7 shows the distribution function for the diffusion operator that acts over one direction z.

6 CONCLUSIONS

A general approach to Monte Carlo methods for Coulomb collisions is given. The approach is based on a special (quasi-Maxwellian) way of approximation of the Landau equation by Boltzmann equation. The possibility of using the other operators to treat heating is shown. Stochastic modeling of the quasilinear diffusion with the diffusion coefficient dependent on velocity is suggested. The effect of the LFP and the diffusion operators combined action results in the distribution tail enhancement. Obtained results are consistent with the preceding results. The results of the paper can be used (by the splitting scheme in the frame of particle methods) for any inhomogeneous problem.

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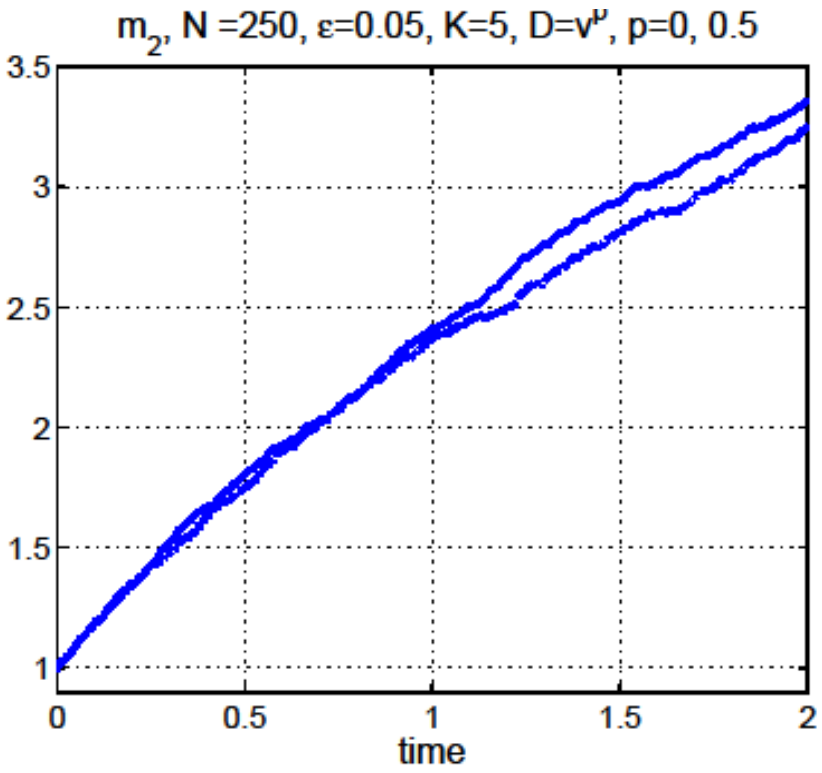

Figure 1: Dependence of the second moment on time $m_2(t)$ (temperature) for two cases $p=0$ and $p=0.5$; K is a number of simulation runs.

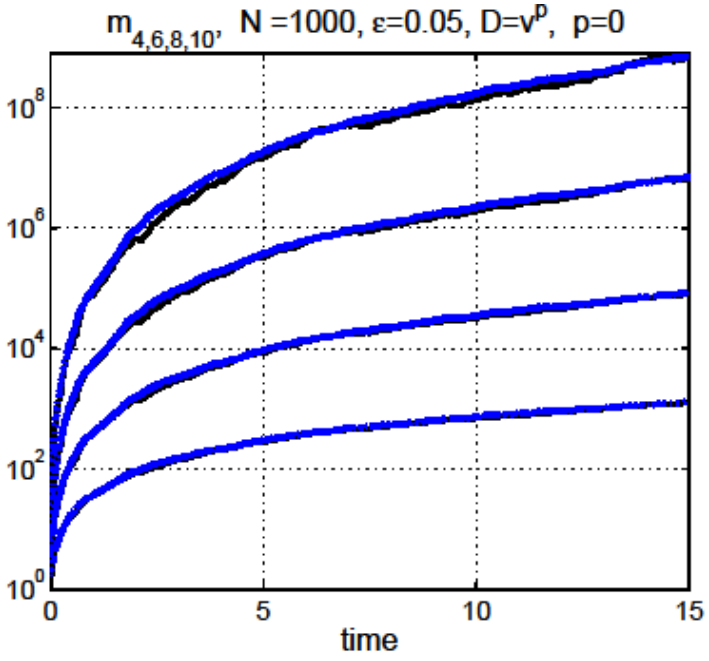

Figure 2: Dependence of the high distribution function moments on time (Eq. (13)) and comparison with the analytical expression Eq.(14) for $p=0$.

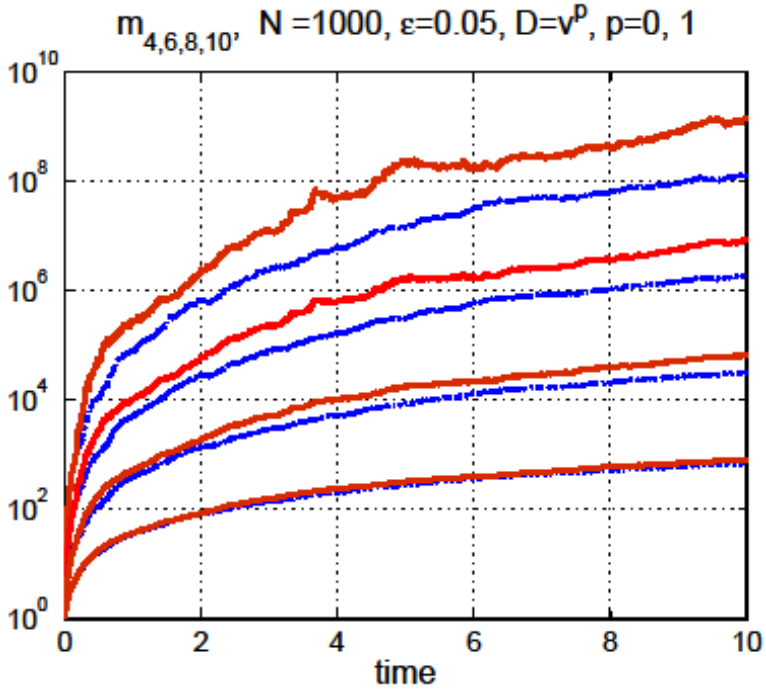

Figure 3: Comparison of the moments $M_{4,6,8,10}(t)$ for $p=0$ - dotted lines and $p=1$- solid lines.

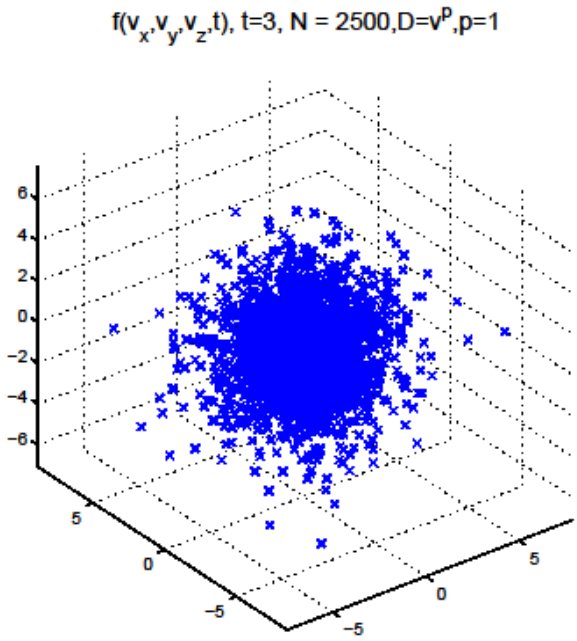

Figure 4: Distribution function for *p=1, t=3.*

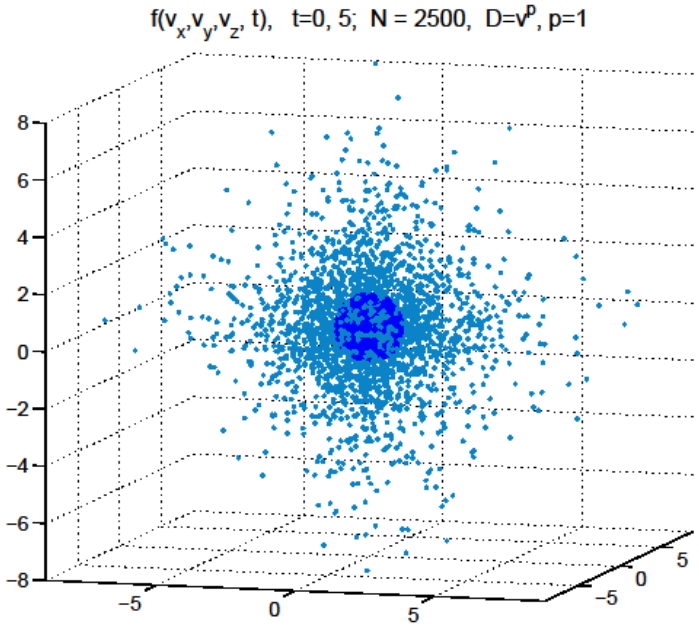

Figure 5: Distribution function for *p=1, t=0* and *t=5.*

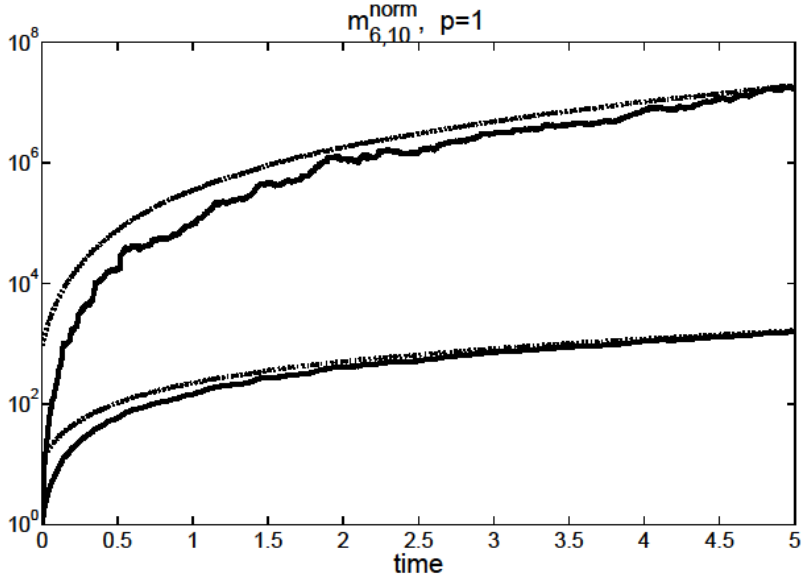

Figure 6: Comparison of the numerical moments $M_{6,10}$ with the analytical moments $m_{6,10}$ for the distribution function (12) normalized on the second moments M_2, m_2, consequently, for the degree *p=1*.

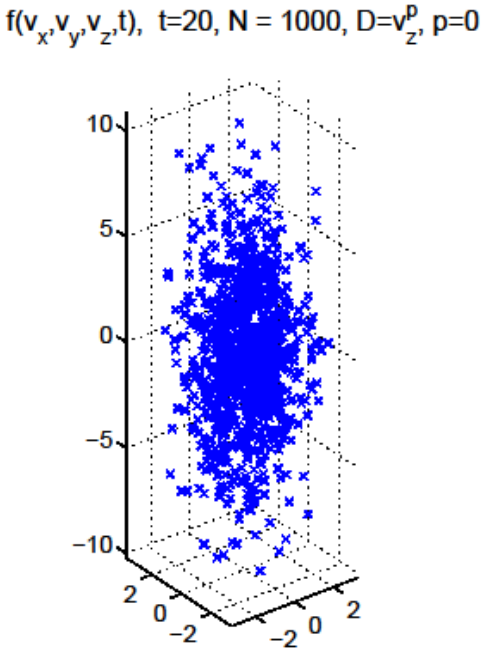

Figure 7: Distribution function for *p=0, t=20* and the diffusion operator acting in *z* direction.

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