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VALUATION OF BARRIER COST OF LOAN CAPITAL TO EFFECTIVE IMPLEMENTATION OF INVESTMENT PROJECT

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Summary. We consider the situation of investment borrowed funds. The barrier discount rates of the projects are calculated from the certain financial flows. According to the results of modeling are makes and analyzed the project barrier rate and the risk of paying capacity dependence on the quantitative and temporal structure of income.

1 INTRODUCTION

Development and implementation of investment strategy is important in the management and strengthening the company's market position in the long term. To form the investment policy of the company are necessary measurement and analysis of of investment projects. The urgency of this line of research follows from its practical importance for the development of investment activity in manufacturing.[1,2]

Successful implementation of the evaluation procedure the efficiency of investment depends on degree of compliance the described situation and the assigned tasks. The variety situations require modification of existing procedures. A lot of attention in the investment projects is given to source of funding.

In the case of leverage, the investor analyzes the situation and predicts future expected returns on investment, which will be used to cover the loan. However, the conditions of the project may get worse, so that may disrupt the schedule of repayment of loan. The company risks losing its ability to pay as a result of possible non-return of leverage in this situation. Therefore, at the stage of making a decision about investing company management is important to know the limit rate, under which you can borrow, given the possible risk of loss of ability to pay.

Usually to assess the effectiveness of the project the interest rate on credit and the internal rate of return (IRR) are chosen empirically based on additional information.

There is a nonlinear relationship between IRR (v) and the rate on credit (i) in case of using only borrowed current assets for investment. In this paper, an attempt not to choose, but to calculate the maximum possible rate i, under which the company can take credit for investment. The basis of this approach is the assumption of equality of IRR (v) and interest rates on credit (i), v = i. Using the equality $v = i$ leads to the need to consider non-linear equation which solved by iterative methods. It should be noted that the equation $v = i$ is a limit relation that indicates the maximum interest rate on the loan for which the investment project is successful. This assumption can be used only under certain conditions.[3,4]

2 ECONOMIC-MATHEMATICAL DEFINITION AND NUMERICAL SOLUTION OF THE PROBLEM

We consider the situation under which the company makes full use borrowed money to invest. We believe that the cash flows of the project is discrete. The calculation period consists of four stages, each of which is equal to one year ($T = 4$). The project provides a one-time investment in a zero period $t = 0$, which may be related, for example, with the purchasing of equipment. For the purpose of investing is used borrowed funds only, which the company receives at the beginning of the period $t = 0$. Disburse a loan is made at the expense of profit. The values of capital repayment and income are presented in Table 1.

Period **t**	Value of capital repayment $\boldsymbol{d_t}$ (m. u.)	Income $\boldsymbol{c_t}$ (m. u.)
0	0	0
1	30 000	50 000
2	20 000	10 000
3	0	10 000

Table 1.

The project provides an opportunity to make a deposit and use of income remaining after payments of the loan to finance the project in the following steps. At the same time the interest rate on deposits coincides with the rate on the loan i. Changing the size of the loan and investments on deposit does not affect the amount of taxes paid.

The principal amount of debt to be paid over two periods - the first and second, the amounts of principal repayment d_t ($t = 0,..T-1$) are known. A delay in the payment of interest on the loan is not, so that the payments start at zero period.

Time structure of financial flows is shown in Fig. 1 as a timeline. Flows of investment returns, marked on the diagram as c_t, is positive and assimilated by the end of the period. The sums of c_t include all the necessary payments.

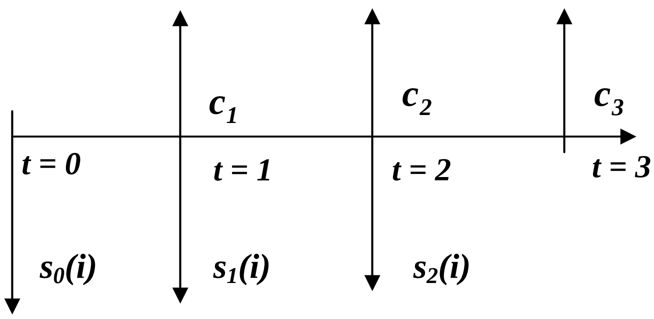

Fig.1. Time diagram of financial flows on the project.

Negative flows of payments on the loan indicated in the diagram as a $s_t(i)$ and consist of the principal amount of debt d_t and interest on the loan $p_t(i)$, which depend on the rate on the loan i, so $s_t(i) = d_t + p_t(i)$. The amount of interest in period t will be $p_t(i) = i\sum_{k=t}^{T-1} d_k$, and payments for periods

$$s_t(i) = d_t + p_t(i) = d_t + i\sum_{k=t}^{T-1} d_k \text{ , } t = 0,1,2,3.$$

To substantiate the possibility of loan repayment is necessary to determine the rate on the loan $0 < i < i_{max}$, which will ensure the effectiveness of the project for the company.

To determine the rate on the loan i, satisfying the conditions stated in the problem and ensuring the effectiveness of the project for the company, we make the assumption that the internal rate of return on the project (IRR) v is equal to the required rate on the loan i, $v=i$. This assumption is possible under the following conditions formulated in [3], which are performed in the assigned task:

1) the project is funded only by the credit;

2) the debt with interest is allowed to return during all the billing period;

3) income after payment of the loan if they occur at some period, shall be made a deposit and can be used to finance the project in the following periods; at the same time the interest rate on deposits is equal to the rate on loans;

4) changes in the size of received loans and investments on deposit does not affect the amount of taxes paid.

The equality $v=i$ makes it possible to find a rate on the loan as the root of a nonlinear equation of profitability, which is made under the condition that the net present value (NPV) of the project equal to zero. Generation of this equation requires the determination of the present value of cash flows of the project by the time $t=0$.[2, 5, 7]

The present value of negative cash flow:

$$S(i,v) = \sum_{t=0}^{T-1} s_t(i)(1+v)^{-t} = \sum_{t=0}^{T-1}\left(\frac{1}{(1+v)^t} d_t + \frac{i}{(1+v)^t}\sum_{k=t}^{T-1} d_k\right), \qquad (1)$$

positive cash flow:

$$C(v) = \sum_{t=0}^{T-1} c_t(1+v)^{-t} \text{ ,} \qquad (2)$$

where v and i - respectively, the unknown internal rate of return of the project and the unknown rate on the loan.

Equality to zero net present value ($NPV=0$) the project involves the equality of the current value of the positive and negative cash flows. Form the equation

$$C(v) = S(i,v), \qquad (3)$$

Taking into account the equality $v = i$ and (1,2) the equation takes the form

$$\sum_{t=0}^{T-1} \frac{c_t}{(1+i)^t} = \sum_{t=0}^{T-1} \left(\frac{1}{(1+i)^t} d_t + \frac{i}{(1+i)^t} \sum_{k=t}^{T-1} d_k \right) \tag{4}$$

The assumption that $v = i$ makes it possible to find a rate on the loan in the form of the root of the nonlinear equation (4). When you select the root of the nonlinear equation (4) need to be careful. A necessary condition for the root of the equation (4) is the internal rate of return of the project, according to [3, 4], is its uniqueness and positivity. Quantity and signs of the roots of the equation (4) can be determined graphically. To do this, we transform equation (4) to the form

$$\sum_{t=0}^{T-1} \frac{c_t}{(1+i)^t} - \sum_{t=0}^{T-1} \left(\frac{1}{(1+i)^t} d_t + \frac{i}{(1+i)^t} \sum_{k=t}^{T-1} d_k \right) = 0 \tag{5}$$

and introduce the notation

$$f(i) = \sum_{t=0}^{T-1} \frac{c_t}{(1+i)^t} - \sum_{t=0}^{T-1} \left(\frac{1}{(1+i)^t} d_t + \frac{i}{(1+i)^t} \sum_{k=t}^{T-1} d_k \right) \tag{6}$$

Localization of the roots is shown in Fig.2.

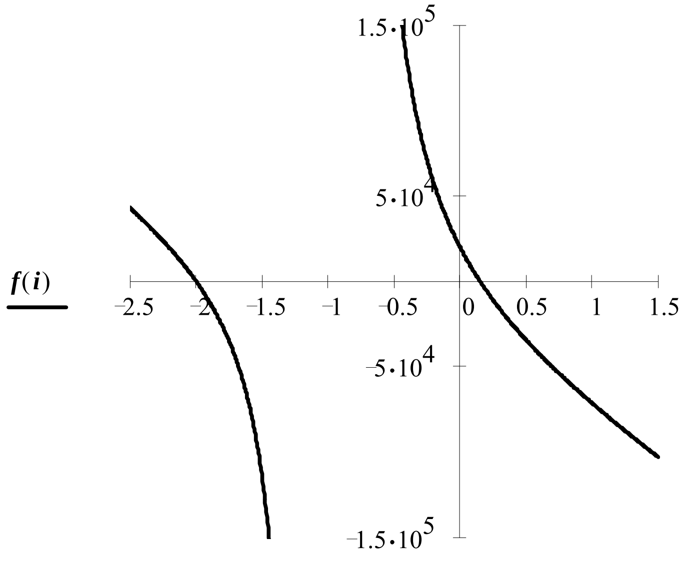

Fig. 2. Localization of the roots of nonlinear equation (5).

According to the graphical representation, equation (5) has two roots: $i_1 \in [-2,5 \div -1,5]$ and $i_2 \in [0 \div 0,5]$. Because the negative root has no economic meaning, it is excluded from consideration. Of interest is the root of i_2. As you can see, the equation has a unique positive root i_2, calculating that we find the IRR of the project v, and by the conditions of equality $v = i$, also a barrier rate on the loan i_{max}.

The nonlinear equation (5), was solved by Newton iteration, by which reduce equation to a convenient form for the solution.[6, 9] We introduce the change

$$x = 1 + i \tag{7}$$

After a simple transformation and change (7) equation (5) takes the form

$$\sum_{t=0}^{T-1} x^{-t}\left(c_t - d_t - (x-1)\cdot\sum_{k=t}^{T-1} d_k\right) = 0 \tag{8}$$

To solve equation (8) all of its parameters must be defined. By the condition $T = 4$, and the values of the coefficients c_t and d_t are shown in Table 1. According to the localization taking into account change (7) is sought root $x \in [1 \div 1{,}5]$. We choose the initial approximation of the root of this segment: $x_0 = 1$. We denote the left-hand side of equation (8) as $f(x)$ and find the derivative of this function

$$f'(x) = -\sum_{t=0}^{T-1}\left[\frac{t}{x^{t-1}}\left(c_t - d_t - (x-1)\sum_{k=t}^{T-1} d_k\right) + \frac{1}{x^t}\sum_{k=t}^{T-1} d_k\right] \tag{9}$$

Obvious that for any $x \in [1 \div 1{,}5]$ $f'(x) \neq 0$, that is, the desired root is simple. We construct, using Newton's method, an iterative procedure for solving equation (8)

$$x_{n+1} = x_n + \frac{\sum_{t=0}^{T-1} x_n^{-t}\left(c_t - d_t - (x_n-1)\cdot\sum_{k=t}^{T-1} d_k\right)}{-\sum_{t=0}^{T-1}\left[\frac{t}{x_n^{t-1}}\left(c_t - d_t - (x_n-1)\sum_{k=t}^{T-1} d_k\right) + \frac{1}{x_n^t}\sum_{k=t}^{T-1} d_k\right]}, \tag{10}$$

where n = 0,1,2… – iteration number.

Starting with $x_0 = 1$, we iterate. The criterion for the end of the iterative process is the inequality

$$|x_{n+1} - x_n| < \varepsilon, \tag{11}$$

where ε – miscalculation, which is for solving was adopted $\varepsilon = 10^{-4}$.

A result of solving of equation (8) found the root of $x^* = 1{,}151$, at which the internal rate of return of the project is equal to $0{,}151$.

3 RESULTS OF MODELING

Project internal rate of return $v = 0{,}15$ was calculated by solving the equation (8). Value $i_{max} = 0{,}15$ is the limiting value of the rate on the loan for an investment project (in view of the equality $v=i$). Net present value $NPV(i)$ will be negative for any value rate if $i > 0{,}15$ (Fig. 3). Getting a credit at interest $i > 0{,}15$ will cause the risk of solvency .

The value of the barrier rate on the loan for the company was calculated from the known financial flows associated with the project (Table 1). During the project may occur worsening conditions, for example a change in the amounts and periods of income. We analyze the effect quantitative and time structure of the amounts expected return and principal on value of the barrier rate. To do this, using the numerical solution of the model (8) we make a forecast of effects of several variants of changing the project.

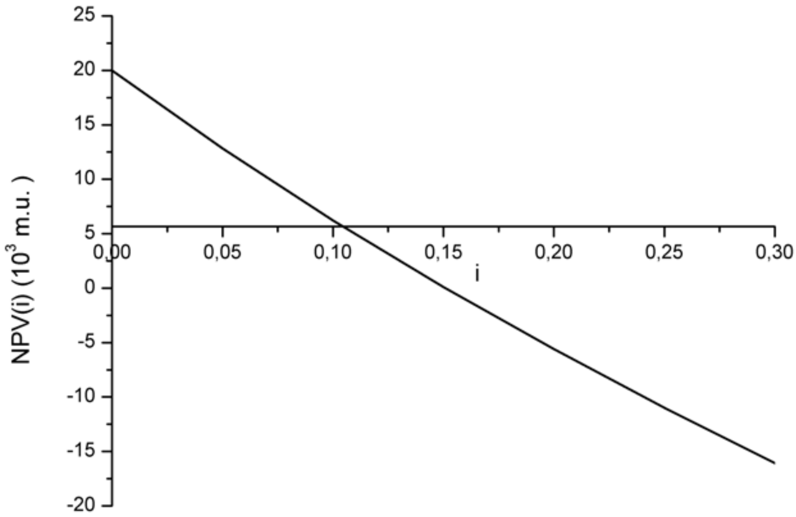

Fig. 3. Net present value *NPV(i)*.

In the first modification of the terms of the project changed the temporal structure of principal payments. Payments of principal d_t changed so that $\sum_t d_t = 50000$, the temporal structure of income c_t has not changed. Calculations showed that the barrier rate i, with different redistribution of amounts of principal of periods does not change, so $i_{max} = 0,15$.

The second modification of the terms of the project changed the time structure of investment income. To investigate the influence of time structure and values of the amounts expected return on the barrier rate and yield of the project were considered conditions of the original project, and two variants of modification, which are presented in Table 2. In the row "variant 1" presents the original project, in the rows "variant 2" and "variant 3" - modifications of the original project. The time structure and the value of payments of principal and were not changed and are not shown in the table. The calculated barrier rates are also presented in Table 2.

Variant k	c_0 (m.u.)	c_1 (m.u.)	c_2 (m.u.)	c_3 (m.u.)	Discount rate i
1	0	50000	10000	10000	0,15
2	0	10000	10000	50000	0,099
3	30000	20000	10000	10000	0,193

Table 2.

The modeling results are presented in Figures 4, 5, 6. Figure 4 shows the areas of return project variants dependence on discount rate i. Areas of return are the areas between the functions of the discounted income $C_k(i)$ and payments on the loan $S(i)$ for $0 \le i \le 1$ for each variant of the project, which are embedded curvilinear triangles. The intersection of $C_k(i)$ and $S(i)$ occurs when the discount rates $i_1 = 0,15$, $i_2 = 0,099$, $i_3 = 0,193$, which are barrier for each of the variants for the project. Barrier rates marked by dashed lines. As can be seen, with a change in the time structure of income profitability of projects has changed. Redistribution of income in the principal amount of the last period (variant 2) reduce the barrier rate and profitability of the project to $i_2 = 0,099$. Such modification reduces the profitability of initial project area by 34%. This variant in Figure 4 corresponds to the smallest area of return. Acquisition of principal income of zero and first periods (variant 3) will increase the barrier rate the initial project to $i_3 = 0,193$. Such modification increases the income of initial project area by 28.7% (Fig. 4).

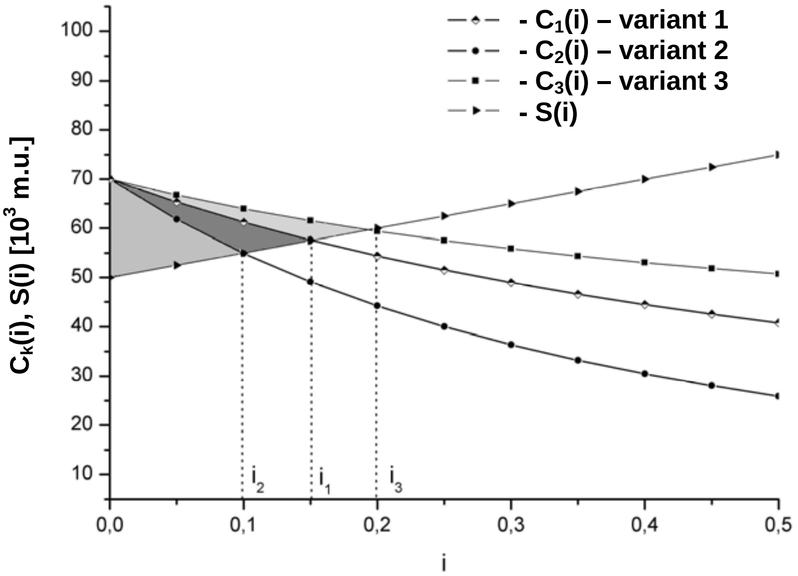

Fig. 4. The dependence of profitability project variants on the percentage of loan *i*.

When planning the investment process is important to know how to influence the distribution of the principal amount of income in different periods of the project on its effectiveness. Such information will allow for preparation for the project to analyze the different variants its implementation.

According to the results of modeling have been made the dependencies of the barrier rate $i(c_t)$ of the maximum possible amount of income c_t, varying in the range $(30\,000 \div 70\,000)$, m.u. at different periods $t = 0, 1, 2, 3$ (Fig. 5).

The greatest influence on the barrier rate has income earned in the period $t = 0$. The effect of income $c_t = 30\,000$ m.u. in the initial period is the highest $i(c_0) = 0,18$. Getting the same income in later periods greatly reduces the barrier rate. In the period t_1 - 29% in the period t_2 - 38%, and in the period t_3 - 40%.

Increasing the amount c_0 up to 70 000 m.u. contributes to the proportional (≈ 55%) growth the barrier rate up to $i(c_0) = 0,4$ (the curve corresponding to the period $t = 0$ in Fig. 5). When you receive the maximum income in the period $t = 1$, you can expect a much smaller growth the barrier rate and the effectiveness of project, it varies from $i(c_1) = 0,13$ when $c_1 = 30\,000$ m.u. to $i(c_1) = 0,18$ when $c_1 = 70\,000$ m.u. In the case where the maximum income will be received in the period $t = 2$, the growth the barrier rate will be only 8%. The barrier rate $i(c_2)$ in the period $t = 2$ varies from 0,11 (at 30 000 m.u.) to 0,119 (at 70 000 m.u.). In the case where the maximum income received in period $t = 3$, you can see the loss of efficiency and increase the amount of income c_3. When $c_3 = 30\,000$ m.u. barrier rate is $i(c_3) = 0,108$, and if $c_3 = 70\,000$ m.u. – $i(c_3) = 0,088$. The efficiency is reduced by 2%.

Thus, the variant with the highest possible income $c_0 = 70\ 000$ m.u. in period $t = 0$ is the most effective, the barrier rate - the highest and the risk of solvency losing due to possible non-repayment of loan is practically reduced to zero, $r(c_0) = 0$, because after receiving the full amount of income in the initial period worsening conditions will not affect the payment of debt.

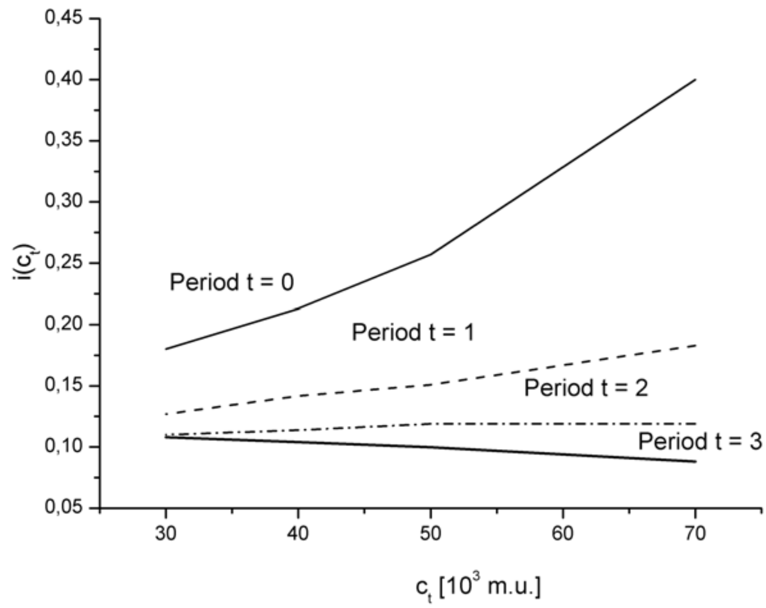

Fig. 5. The dependence of the barrier rate on the loan on quantitative and time structure of income for the period of the project.

As shown by calculations, any changes to the quantitative and time structure of income leads to a decrease in barrier rate $i(c_t)$ and profitability of the project compared to the risk-free barrier rate $i(c_0) = 0,4$. It can be assumed that the reduction the barrier rate and effectiveness of the project is due to the risk of changes in the quantitative and time structure of income $r(c_t)$.

The dependence of the maximum barrier risk-free rate, level of risk and the barrier rate on the change in the conditions of the project can be written as

$$i(c_0) = (1 + r(c_t)) \cdot (1 + i(c_t)) - 1, \qquad t = 1,2,3\,, \tag{12}$$

where $r(c_t)$ – the level of risk reduction the amount of profit c_t in t period.

If $r(c_t) = 0$, the equation (12) takes the form $i(c_t) = i(c_0),$), thus the barrier rate $i(c_t)$ will be maximum. From (12) we can calculate the level of risk $r(c_t)$ associated with a change in the quantitative and time structure of income

$$r(c_t) = \frac{i(c_0) - i(c_t)}{i(c_t) + 1}\,.$$

Figure 6 shows dependence the level of risk $r(c_t)$ of solvency for planning maximum amount of income ($c_t = 30\ 000 \div 70\ 000$ m.u. at different periods t. The greatest influence on the value of the barrier rate have an income of zero and first.

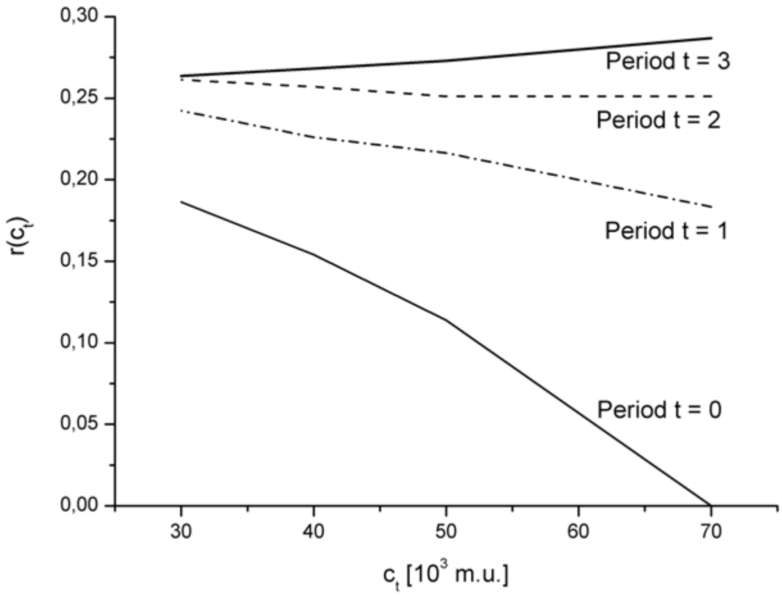

Fig. 6. Risk of solvency dependence on the quantitative and time structure of income by periods.

Risk curves increase significantly with a decrease in the amount of income in each of these periods. Decrease income in a zero period causes an increase in the level of risk from 0 to 0,19 ($r(c_0) = 0 \div 0,19$), and in the first period – from 0,18 to 0,24. Planning weighty amounts of income in the second and third periods of the project associated with the greatest risk of loss of solvency. If the high values ($r(c_2) = 0,25 \div 0,26$, $r(c_3) = 0,26 \div 0,29$), then the change in risk due to a decrease in revenues in these periods is small. Thus, in the second and third periods, the level of risk is more dependent on the period of income than the amount. The level of risk $r(c_t)$ can be regarded as the amount by which is necessary to raise the barrier rate to compensate the risk of partial repayment of the loan, thus increasing the stability of the project.

4 CONCLUSION

One of the methods evaluating the effectiveness of investment borrowings is considered. Formulated the mathematical model of the effectiveness of investment borrowings.[7, 8] By well-known financial flows associated with the project, defined the barrier (limit) discount rate for using borrowed funds. With the results of modeling predicted changes in the terms the considered project. The first variant of modifications related to changes in the time structure payments of principal has shown that such changes do not affect the magnitude of the barrier rate. The second variant of modifications associated with a change in the quantitative and the time structure of income for the project. As a result of modeling obtained data showing the degree of influence each period of income by the amount of barrier rates and risk losing the company's solvency as a result of changes in the quantitative and the time structure of income.

It is shown that the barrier is the highest discount rate if the income is received in the initial period of the project. So, the increase in the amount of income in a zero period (30

000 ÷ 70 000 m.u.) causes the growth the barrier rate around 55%. At the same change in the amount of income in the first period the barrier rate will increase by only 27%, in the second period – by 8%. Increase the amount of income in the third period, causing a negative growth the barrier rate - 2%.

It is analyzed, how period of income affects the level of risk. It is shown that the zero and first periods, the level of risk is sensitive to amounts of income, with a decrease in the amount of income in each of these periods the risk increases significantly (≈ 20 times). Redistribution of the principal amount of income in the second and third periods increases the level of risk in about 30 times, regardless of the amount of income.

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