MODELING OF SHOCK-WAVE PROCESSES IN ALUMINUM UNDER THE ACTION OF A SHORT LASER PULSE

A. YU. SEMENOV¹,², I. A. STUCHEBRYUKHOV¹ AND K. V. KHISHCHENKO³,²,⁴

¹ Prokhorov General Physics Institute of the Russian Academy of Sciences
Vavilova 38, 119991 Moscow, Russia

² Moscow Institute of Physics and Technology
Institutskiy Pereulok 9, 141701 Dolgoprudny, Moscow Region, Russia

³ Joint Institute for High Temperatures of the Russian Academy of Sciences
Izhorskaya 13 Bldg 2, 125412 Moscow, Russia

⁴ South Ural State University
Lenin Avenue 76, 454080 Chelyabinsk, Russia

*Corresponding author. E-mail: konst@ihed.ras.ru

DOI: 10.20948/mathmontis-2021-50-10

Summary. A hydrodynamic model of shock-wave processes in a material under the action of a short high-intensity laser pulse is considered. The simulation is carried out for the case of an aluminum target 90 µm thick, irradiated by a laser pulse with a duration of 70 ps and a maximum intensity of 14.7 TW/cm². In the corresponding laboratory experiment, on the rear side of the target after irradiation, a spall of a part of the material is recorded at a depth of 10 ± 1 µm. Calculation of the time dependence of the pressure and density of aluminum in the spall plane makes it possible to determine the tensile strength of aluminum at a high strain rate.

1 INTRODUCTION

The action of a short high-intensity laser pulse upon a target makes it possible to study the properties of the target material under shock-wave loading at a high strain rate [1–9]. Numerical modeling of such a process [2, 3, 6, 10–16] provides additional possibilities for the interpretation of the obtained measurement results.

In this work, an example of a laboratory experiment on the action of a 70 ps laser pulse on an aluminum target is given. A description of the hydrodynamic model for the propagation of shock compression waves and adiabatic unloading along the target is presented. The results of modeling are presented and a conclusion is made about the magnitude of the spall strength of aluminum at the considered strain rate.

2010 Mathematics Subject Classification: 74J40, 76L05, 82D20, 82D35.

Key words and phrases: laser pulse action, aluminum, shock wave loading, adiabatic release, spall, tensile strength, high strain rate.
2 EXPERIMENT

The experiment was carried out on a Kamerton-T facility based on a neodymium glass laser (wavelength \( \lambda = 0.527 \ \mu m \)) [8, 10, 11]. A pulse with duration \( \tau = 70 \) ps and energy \( E_l = 3.57 \) J was focused into a spot 0.63 mm in diameter on the surface of a 90-\( \mu \)m-thick aluminum target. Taking into account the measured dependence of the laser radiation intensity on time, the maximum intensity of this pulse is estimated to be \( I_0 = 14.7 \) TW/cm\(^2\).

The result of the action of such a pulse is the formation of a spall of a part of the material on the rear side of the target. The spall occurred at a distance of 10 \( \pm 1 \) \( \mu \)m from the rear surface; the diameter of the spalled plate is 0.66 mm.

3 HYDRODYNAMIC MODEL

The system of hydrodynamic equations for the one-dimensional case under consideration has the following form [17]:

\[
\begin{aligned}
\frac{\partial}{\partial t} U + \frac{\partial}{\partial x} F &= 0, \\
U &= \begin{pmatrix}
\rho \\
\rho u \\
\rho v \\
\rho w \\
e
\end{pmatrix}, \\
F &= \begin{pmatrix}
\rho u \\
\rho u^2 + P \\
\rho uv \\
\rho uw \\
(e + P)u
\end{pmatrix},
\end{aligned}
\]

where \( t \) is the time coordinate; \( x \) is the spatial coordinate; \( \rho \) is the density of the material under consideration; \( P \) is the pressure; \( e \) is the full energy density,

\[
e = \rho E + \frac{1}{2} \rho (u^2 + v^2 + w^2),
\]

\( E \) is the specific internal energy; \( u \) is the particle velocity along the \( x \)-axis; \( v = 0 \) and \( w = 0 \) for the case.

In quasilinear non-conservative form, the system of equations (1) can be written as follows:

\[
\frac{\partial}{\partial t} U + A \frac{\partial}{\partial x} U = 0,
\]
where

\[
A = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 \\
-u^2 + \theta b & 2u - ub & -vb & -wb & b \\
-uv & v & u & 0 & 0 \\
-uw & w & 0 & u & 0 \\
-u + u\theta b & h - u^2b & -uvb & -uwb & u + ub
\end{pmatrix},
\]  

\[h = \frac{e + P}{\rho}, \quad \theta = q^2 - \frac{e}{\rho} + \left(\frac{\partial P}{\partial \rho}\right)_E b, \quad q^2 = u^2 + v^2 + w^2, \quad b = \frac{(\partial P/\partial E)_{\rho}}{\rho}. \quad (6)
\]

One can write matrix \( A \) in the form \( A = \Omega\Lambda\Omega^{-1} \), where

\[
\Omega = \begin{pmatrix}
1 & 0 & 0 & 1 & 1 \\
u - c & 0 & 0 & u & u + c \\
v & 1 & 0 & v & v \\
w & 0 & 1 & w & w \\
h - uc & v & w & h - c^2b^{-1} & h + uc
\end{pmatrix}, \quad \Lambda = \begin{pmatrix}
u - c & 0 & 0 & 0 & 0 \\
0 & u & 0 & 0 & 0 \\
0 & 0 & u & 0 & 0 \\
0 & 0 & 0 & u & 0 \\
0 & 0 & 0 & 0 & u + c
\end{pmatrix}, \quad (7)
\]

\[
\Omega^{-1} = \frac{b}{2c^2} \begin{pmatrix}
\theta + uc\rho^{-1} & -u - cb^{-1} & -v & -w & 1 \\
-2v\rho^{-1} & 0 & 2c^2b^{-1} & 0 & 0 \\
-2w\rho^{-1} & 0 & 0 & 2c^2b^{-1} & 0 \\
2h - 2q^2 & 2u & 2v & 2w & -2 \\
\theta - uc\rho^{-1} & -u + cb^{-1} & -v & -w & 1
\end{pmatrix},
\]  

\[
det\Omega = \frac{2c^3}{b}, \quad c = \sqrt{\left(\frac{\partial P/\partial \rho}_{E} + \frac{P}{\rho^2}\left(\frac{\partial P/\partial E}_{\rho}\right)\right)} \quad (9)
\]

Here, \( c \) is the adiabatic sound velocity. Using values \( c \) and \( h \), one can obtain

\[
\theta = q^2 - h + c^2b^{-1}. \quad (10)
\]

The system of equations of motion (1) is closed by the equation of state of the target material in the form of a function

\[
P = P(\rho, E), \quad (11)
\]

which is taken according to the model [18–20].
4 SOLUTION METHOD

With the use of the formulas for matrices from the previous section, the system of equations (1) and (11) can be solved by the Courant–Isaacson–Rees method [21]. The difference scheme of the method is as follows [17]:

\[
\frac{U^{k+1}_j - U^k_j}{\Delta t} + \frac{F_{j+1/2}^{k+1/2} - F_{j-1/2}^k}{\Delta x} = 0,
\]

\[
F_{m+1/2} = \frac{1}{2} (F^k_m + F^k_{m+1}) + \frac{1}{2} |A|_{m+1/2}^k (U^k_m - U^k_{m+1})
\]

for \( m = j - 1 \) and \( j \). Here, integer subscripts denote the values of the function at the centers of discrete grid cells in space, and half-integer ones—at the boundaries of the cells; \( \Delta x \) is the step of a uniform grid in space; \( \Delta t \) is the time step;

\[
|A| = \Omega |\Lambda| \Omega^{-1},
\]

\[
|\Lambda| = \begin{pmatrix}
|u - c| & 0 & 0 & 0 & 0 \\
0 & |u| & 0 & 0 & 0 \\
0 & 0 & |u| & 0 & 0 \\
0 & 0 & 0 & |u| & 0 \\
0 & 0 & 0 & 0 & |u + c|
\end{pmatrix}.
\]

Matrix (15) can be represented as a sum of three matrices with multipliers:

\[
|\Lambda| = |u| \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix} + \alpha \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix} + \gamma \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix},
\]

where \( \alpha = |u - c| - |u|; \gamma = |u + c| - |u| \).

Denoting \( \Delta U = U_m - U_{m+1} \) (with elements \( \Delta U = U_m - U_{m+1} \)) and using (14) and (16), one can obtain

\[
|A| \Delta U = |u| \begin{pmatrix}
\Delta \rho \\
\Delta (\rho u) \\
\Delta (\rho v) \\
\Delta (\rho w) \\
\Delta e
\end{pmatrix} + \alpha (f + g) \begin{pmatrix}
1 \\
0 \\
0 \\
0 \\
0
\end{pmatrix} + \gamma (f - g) \begin{pmatrix}
1 \\
0 \\
0 \\
0 \\
0
\end{pmatrix},
\]

where \( \alpha = |u - c| - |u|; \gamma = |u + c| - |u| \).
where
\[
 f = \frac{b}{2c^2}[\theta \Delta \rho - u \Delta (\rho u) - v \Delta (\rho v) - w \Delta (\rho w) + \Delta e], \quad g = \frac{1}{2c}[u \Delta \rho - \Delta (\rho u)]. \quad (18)
\]

Taking into account relations (6) as well as \( \Delta P \approx (\partial P/\partial \rho)E \Delta \rho + (\partial P/\partial E) \rho \Delta E \) and \( \Delta (\rho u) \approx \rho \Delta u + u \Delta \rho \), it is possible to obtain approximate expressions for the factors \( f \) and \( g \):
\[
 f \approx \frac{1}{2c^2} \Delta P, \quad g \approx -\frac{\rho}{2c} \Delta u. \quad (19)
\]

So, in (13), one can use
\[
 |A|_{m+1/2}^k (U_m^k - U_{m+1}^k) = \left( \begin{array}{c} |u|_{m+1/2}^k (\rho_m^k - \rho_{m+1}^k) + \beta_{m+1/2}^k \\ |u|_{m+1/2}^k (\rho v_m^k - \rho u_{m+1}^k) + [\beta u - \delta c]_{m+1/2}^k \\ |u|_{m+1/2}^k (\rho w_m^k - \rho v_{m+1}^k) + [\beta v]_{m+1/2}^k \\ |u|_{m+1/2}^k (\rho v_{m+1/2}^k - \rho w_{m+1}^k) + [\beta v]_{m+1/2}^k \\ |u|_{m+1/2}^k (\rho w_{m+1/2}^k - \rho v_{m+1}^k) + [\beta v]_{m+1/2}^k \\ |u|_{m+1/2}^k (\rho v_{m+1/2}^k - \rho w_{m+1}^k) + [\beta v]_{m+1/2}^k \end{array} \right), \quad (20)
\]
\[
 \beta_{m+1/2}^k = [\alpha (f + g) + \gamma (f - g)]_{m+1/2}^k, \quad \delta_{m+1/2}^k = [\alpha (f + g) - \gamma (f - g)]_{m+1/2}^k, \quad (21)
\]
\[
 f_{m+1/2}^k = \left( \begin{array}{c} 1 \\ 2c \end{array} \right)_{m+1/2}^k (\rho_m^k - \rho_{m+1}^k), \quad \delta_{m+1/2}^k = \left[ \frac{\rho}{2c} \right]_{m+1/2}^k (u_m^k - u_{m+1}^k). \quad (22)
\]

At the initial moment of time, the entire target was divided in thickness into cells of the same size, which were numbered from \( j = 1 \) to \( N_j = 1000 \). This gives the step of the grid in space \( \Delta x = 0.09 \mu m \). The step in time was chosen from the condition \( \Delta t \leq \xi \Delta x / \max(|u| + c) \), where \( \xi = 0.1 \).

At each time step, the boundaries of the cells shifted with a certain velocity \( D \), which is the particle velocity \( u \) for the case under consideration:
\[
 x_{m+1/2}^{k+1} = x_{m+1/2}^k + D_{m+1/2}^k \Delta t. \quad (23)
\]

The construction of a difference scheme for such a moving grid is based upon the system of hydrodynamic equations in integral form:
\[
 \int_L (Udx - Fdr) = 0, \quad (24)
\]
where \( L \) is the contour that bounds the region of integration on the coordinate plane \((x, t)\). As this contour \( L \), it is suitable to take a difference cell with number \( j \) with height \( \Delta x \) and bases \( \Delta x_{j+1}^k \) and \( \Delta x_j^k \), where \( \Delta x_m = x_{m+1/2} - x_{m-1/2} \). Approximating integral equation (24), one can
obtain

\[(U \Delta x)_{j+1}^k - (U \Delta x)_j^k + F_{j+1} \Delta t - F_{j-1} \Delta t = 0,\]  \hspace{1cm} (25)

or, instead of (12),

\[
\frac{(U \Delta x)_{j+1}^k - (U \Delta x)_j^k}{\Delta t} + F_{j+1} - F_{j-1} = 0.\]  \hspace{1cm} (26)

A local transition to a coordinate system that moves with constant velocity \(D\) relative to the original system (Galilean transformation) changes the original form of the system of equations (1):

\[
\frac{\partial}{\partial t} U + \frac{\partial}{\partial x} (F - DU) = 0.\]  \hspace{1cm} (27)

In this regard, the flows (13) at the boundaries of the cells change:

\[
F_{m+1/2} = (F - DU)_{m+1/2} = \frac{1}{2} (F_m^k + F_{m+1}^k) - \frac{1}{2} (U_m^k + U_{m+1}^k) D_{m+1/2}^k + \frac{1}{2} |A_D|^{1/2} (U_m^k - U_{m+1}^k), \]  \hspace{1cm} (28)

\[
|A_D| = \begin{vmatrix} \left|u - D - c\right| & 0 & 0 & 0 & 0 \\ 0 & \left|u - D\right| & 0 & 0 & 0 \\ 0 & 0 & \left|u - D\right| & 0 & 0 \\ 0 & 0 & 0 & \left|u - D\right| & 0 \\ 0 & 0 & 0 & 0 & \left|u - D + c\right| \end{vmatrix}, \]  \hspace{1cm} (29)

\[
|\Lambda_D| = \begin{vmatrix} \left|u - D - c\right| & 0 & 0 & 0 & 0 \\ 0 & \left|u - D\right| & 0 & 0 & 0 \\ 0 & 0 & \left|u - D\right| & 0 & 0 \\ 0 & 0 & 0 & \left|u - D\right| & 0 \\ 0 & 0 & 0 & 0 & \left|u - D + c\right| \end{vmatrix}. \]  \hspace{1cm} (30)

5 INITIAL CONDITIONS

The initial values of pressure, density and particle velocity were set constant over the target: \(P = 0.1\) MPa, \(\rho = \rho_0 = 2.712\) g/cm³ and \(u = v = w = 0\). The initial value of specific internal energy was taken according to the used equation of state for aluminum.

6 BOUNDARY CONDITIONS

On the irradiated surface of the target, a pressure profile

\[P(t) = P_a 16^{-(t-t_0)^2 \tau^{-2}} \text{ (for } 0 < t < t_1), \hspace{1cm} P(t) = 0 \text{ (for } t \leq 0 \text{ or } t_1 \leq t) \]  \hspace{1cm} (31)
was set, calculated using approximated dependence of the laser radiation intensity

\[ I_1(t) = I_0 16^{-(t-t_0)^2/\tau^2} \]  

and the scaling relation, which is formulated for the range \( 4.3 < I_0 \leq 1000 \text{ TW/cm}^2 \) [22, 23]:

\[ P_a = P_{a0} (\lambda_I I_0 / \lambda)^{2/3} [A_u / (2Z)]^{3/16}, \]  

where \( P_{a0} = 1.2 \text{ TPa}; \lambda_I = 10^{-2} \text{ µm cm/TW}; A_u \) and \( Z \) are the atomic mass (u) and the atomic number of the target material respectively, \( A_u = 26.98154 \) and \( Z = 13 \) for aluminum. On the rear side of the target, pressure was set equal to zero.

7 SIMULATION RESULTS

The simulation was performed for the case of loading pressure pulse (31) with the magnitude \( P_a = 516 \text{ GPa} \) according to equation (33); \( t_0 = 123 \text{ ps}, t_1 = 246 \text{ ps} \).

Figure 1 illustrates the change in pressure during the propagation of compression and rarefaction waves through the aluminum target. In figure 1(a), one can see that the rarefaction wave follows the shock wave while both move towards the rear side of the target. After the shock wave has reached the rear side, one more rarefaction wave begins to move backward [see figure 1(b)]. When these two rarefaction waves meet, a spalling phenomenon occurs.

Figure 2 shows the calculated pressure and density histories in three planes, which correspond to the initial distances from the rear side of the target 9, 10 and 11 µm.

The curves shown in figure 2(a) allow one to estimate the maximum possible tensile stress \( \sigma_{\text{max}} \) in the sample in the spall plane. The absolute value of the pressure at the minimum on the curve for the plane where the spallation occurred in the experiment is this maximum possible tensile stress. The difference in values in two adjacent planes (for which the initial position differs from the initial position of the spall plane by the value of the error in determining the spall depth), divided in half, gives the average error in determining the maximum possible tensile stress in the sample.

The calculated curves shown in figure 2(b) allow one to estimate the maximum strain rate \( \rho_0 dV/dt = -\rho_0 \rho^{-2} d\rho/dt \) in the spall plane at the stage of tension at negative pressures. Here, \( V = \rho^{-1} \) is the specific volume. Starting from the point of zero pressure, when the sample is stretched, the strain rate decreases monotonically to zero at the point of minimum pressure. Then, with time, the tensile stress decreases, and the strain rate becomes negative (i.e., the density increases with pressure).

In the case under consideration, \( \sigma_{\text{max}} \pm \Delta \sigma_{\text{max}} \approx 7.2 \pm 0.5 \text{ GPa}, \rho_0 dV/dt \pm \Delta(\rho_0 dV/dt) \approx 0.22 \pm 0.01 \text{ ns}^{-1} \) for aluminum.
Figure 1: Pressure in the target at \( t = 3.6, 7.4, 11 \) (a), 12.7, 15.2 and 18 ns (b) along the coordinate axis \( x \), which is perpendicular to the irradiated surface, with the origin at the point of the initial position of this surface before the experiment.
Figure 2: Pressure (a) and density (b) histories in three planes that correspond to the initial distances from the back of the target 9, 10 and 11 µm. The thin vertical lines correspond to the moments of reaching the maximum tensile stress (negative pressure).
8 CONCLUSIONS

Thus, in a laboratory experiment on irradiating a 90-µm-thick aluminum plate with a 70 ps laser pulse with a maximum intensity of 14.7 TW/cm², a spall was obtained at a distance of 10 ± 1 µm from the rear surface of the target. A hydrodynamic model has been developed for the propagation and interaction of shock and release waves in a target under such a pulsed action. As a result of the calculation using the developed model, the maximum possible tensile stress in the sample in the spall plane is determined as 7.2 ± 0.5 GPa and the maximum strain rate at the stage of tension at negative pressures is determined as 0.22 ± 0.01 ns⁻¹.

Acknowledgments: The authors dedicate this article to the memory of their teacher, colleague and friend Igor Kornelievich Krasyuk (07.03.1942–26.05.2020).

This work is financially supported by the Russian Foundation for Basic Research (project No. 20-02-00683).

The paper is based on the proceedings of the XXXVI International Conference on Interaction of Intense Energy Fluxes with Matter, Elbrus, the Kabardino-Balkar Republic of the Russian Federation, March 1 to 6, 2021.

REFERENCES


Received December 20, 2020