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COMPARATIVE ANALYSIS OF THE ACCURACY OF OPENFOAM SOLVERS FOR THE OBLIQUE SHOCK WAVE PROBLEM

A. K. ALEKSEEV[1], A.E. BONDAREV[1*], A.E. KUVSHINNIKOV[1]

[1]Keldysh Institute of Applied Mathematics Russian Academy of Sciences
Moscow, Russia
*Corresponding author. E-mail: bond@keldysh.ru

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Summary. The article is devoted to a comparative assessment of the accuracy for solvers of the OpenFOAM open software package. As a test problem, we consider the classical two-dimensional problem of a supersonic inviscid compressible flow falling on a flat plate at an angle of attack. As a result, an oblique shock wave is formed before the start of the plate. The simulation results for the solvers considered in comparison are compared with the known exact solution. Calculations for all solvers participating in the comparison were carried out with the same setting of the parameters of the incident flow and angle of attack. Special attention was paid to QGDFoam solver, which has controlled dissipative properties. For this solver, within the framework of a general comparison, calculations were carried out with a variation of the parameter that allows controlling dissipative properties. The results of estimates of deviations from the exact solution in various norms for all solvers are given.

1 INTRODUCTION

A comparative assessment of the accuracy and efficiency of numerical methods and algorithms for mathematical modeling of CFD problems has been the subject of special attention of researchers throughout the development of mathematical modeling. Over a long period, a certain set of CFD test problems has developed, and the verification of the efficiency of the developed numerical method was based on testing the method on this set of problems. These aspects are reflected in a fairly large number of reviews, for example, [1, 2].

Currently, the task of comparative assessment of numerical methods accuracy has not lost its relevance. New technical problems and the appearance of new mathematical models entail intensive development of numerical methods. Developed new numerical methods and algorithms are often implemented in the form of solvers integrated into various software packages, both commercial and open. In this process, not always and not all solvers pass a full test on the classical set of test problems. This set can include such well-known problems as: falling an oblique shock wave onto a plate, rarefaction wave, forming a boundary layer on a plate or a smooth curved surface, flow in front of the obstacle, flow in front of a spherically blunt obstacle, flow around a cone, flow behind a ledge, flow in the far wake. Such a set of tests provides testing of a numerical method and its software implementation for mathematical models, describing both inviscid flows and viscous ones.

It should be noted that the analysis of the accuracy of numerical methods in the simulation of discontinuities has been relevant since the main directions and approaches to the simulation of flows with shock waves were formed. Two major directions were formed here - methods for gas-dynamic flows modeling without marking discontinuities and methods where discontinuities were defined as boundaries of a flowfield. A detailed description of these directions can be found in [3,4]. Both directions are characterized by a large number of

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numerical methods implemented in their framework. As examples of approaches without isolating of discontinuities, one can cite papers [5–9].

Methods that use the isolating of discontinuities in the form of the boundaries of the computational domain can be found, for example, in [10–12]. Practical experience of using both approaches has shown that both approaches have both clear advantages and obvious disadvantages. Thus, numerical methods without isolating of discontinuities require to make the computational grid more detailed in the vicinity of discontinuities and, as a result, give a vague picture of the flow structure. In turn, methods that use discontinuity detection encounter problems when modeling rapidly changing structures of shock waves, which leads to the need for a rapid restructuring of the geometry of the computational regions and the introduction of scenario approaches for organizing such a restructuring. Nevertheless, serious attempts are being made to overcome such difficulties in both directions.

For methods without isolating discontinuities, it is possible to note the work [13]. The approach proposed in this paper makes it possible to clearly identify the locations of discontinuities by automating the processing of the computed results. In this approach, gas-dynamic functions are considered as the intensity of the image, and the values of the functions at each point as the elements of the image (pixels). A differential detector is used and a detectable fracture is classified using discrete analogs of gas-dynamic relations performed at the discontinuity. The stated approach does not depend on the specific type of the problem being solved and does not require any a priori information about the flow. As an example of an approach combining both directions, we can mention the method of dynamic adaptation presented in [14-18]. The method is based on the transition to an arbitrary nonstationary coordinate system in which not only grid functions, but also coordinates of grid nodes are unknown. According to [18], this approach allows for calculations using methods without isolating discontinuities with automatic condensing of grid nodes to the solution features, and with explicit selection of moving boundaries and discontinuities when necessary. It should be emphasized that for methods that do not use the selection of boundaries, an analysis of the solution behavior at a discontinuity and an assessment of the accuracy are necessary.

This paper is devoted to comparing the accuracy of solvers of an open software package OpenFOAM [19] at an oblique shock wave and continues the research on the comparative assessment of numerical methods accuracy on classical test problems. At the previous stages of this study, a comparative assessment was made of the accuracy of the OpenFOAM solver group for the task of flow around a cone under the angle of attack. Studies were performed with a variation of the Mach number, the angle of the cone and the angle of attack in wide ranges with the selected step. Thus, a study was performed for a class of problems defined for these determining parameters within the ranges of variation. The results of the comparison with the well-known tabular solution allowed us to construct the dependence of the error on the determining parameters for each solver and to make a comparison for the class of problems in question. The main results are presented in [20–23].

It should be noted that these numerical studies were based on the principles of constructing a generalized computational experiment [24-27]. The construction of such an experiment is based on numerical parametric studies and the solution of optimization analysis problems. Solving such problems implies a multiple solution to the direct problem of numerical modeling of a gas-dynamic process with various input data. The defining parameters of a class of problems, such as the characteristic Mach number, Reynolds number, geometric parameters, etc., vary in certain ranges with a certain partitioning step. As a result, the resulting solution is a multidimensional volume of data. To analyze this volume, modern methods of data analysis and visual analytics are used.

In this paper, a comparison is made for the solvers of the OpenFOAM (Open Source Field Operation And Manipulation CFD Toolbox) open source software package. This is a free software product created for solving problems of hydro and gas dynamics. Widely used in many areas of science and technology, the OpenFOAM package contains a number of solvers with different computational properties. The OpenFOAM package also allows one to develop new solvers on the platform of package. Four solvers participated in the comparison: two standard solvers - *rhoCentralFoam* and *sonicFoam* and two new solvers - *pisoCentralFoam* [28] and *QGDFoam* [29,30]. The last two solvers were developed by teams of Ivannikov Institute for System Programming and Keldysh Institute of Applied Mathematics of the Russian Academy of Sciences. It should be noted that the *QGDFoam* solver has a controllable parameter that allows one to adjust the dissipative properties of the numerical method, which is extremely important in suppressing unwanted oscillations at shock waves. The research in this paper for *QGDFoam* was performed with a variation of this parameter.

Previously, most comparative estimates of numerical methods accuracy in simulating a shock wave were reduced to a comparison of the width of the shock wave spreading zone along a selected line crossing the discontinuity. In this paper, we use error estimates for the entire flow field in the computational domain in different norms.

2 FORMULATION OF THE TEST PROBLEM

In this paper, the classical two-dimensional inviscid problem of modeling an oblique shock wave is used to compare solvers. The general flow scheme is shown in Fig. 1. A supersonic gas flow with Mach number M at an angle β falls on a flat plate. At the beginning of the plate, an oblique shock wave S occurs. This problem is considered within the framework of the Euler system of equations and has an exact analytical solution.

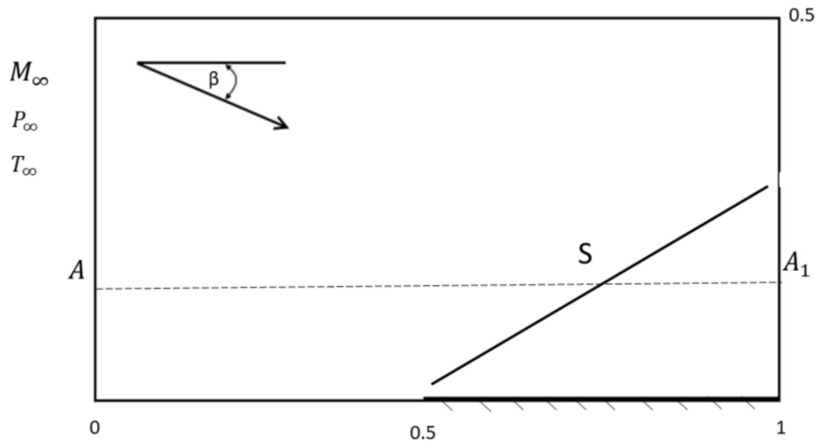

Fig.1. Flow scheme.

At the input boundary, the parameters of the external flow are specified for the Mach number M and a certain value β. On the part of the lower boundary corresponding to a flat plate, a no-flow condition is specified. At the output boundary, we set the derivatives of gas-dynamic functions equal to zero along the normal to the boundary. On the upper boundary for the velocity components the boundary conditions are set similarly to the conditions for the input boundary. For the remaining gas-dynamic functions of the upper boundary the conditions are set similarly to the conditions for the output boundary.

3 THE PROCEDURE OF COMPARISON

The solution of the problem was performed using 4 solvers of the OpenFOAM software package. These solvers were: *rhoCentralFoam*, *sonicFoam*, *pisoCentralFoam*, *QGDFoam*. We give below their brief characteristics.

Solver *rhoCentralFoam* — is based on a central-upwind scheme, which is a combination of central-differential and upwind schemes [31,32]. The essence of the central-upwind flow schemes consists in a special selection of the control volume containing two types of domains: around the boundary points - the first type; around the center point - the second type. The boundaries of the control volumes of the first type are determined by means of local propagation velocities. The advantage of these schemes is that, using the appropriate technique to reduce the numerical viscosity, it is possible to achieve good solvability for discontinuous solutions — shock waves in gas dynamics, and for solutions in which viscous phenomena play a major role.

Solver *sonicFoam* is based on the PISO algorithm (Pressure Implicit with Splitting of Operator) [33]. The basic idea of the PISO method is that two difference equations are used to calculate the pressure for the correction of the pressure field obtained from discrete analogs of the equations of moments and continuity. This approach is due to the fact that the velocities corrected by the first correction may not satisfy the continuity equation, therefore, a second corrector is introduced which allows us to calculate the velocities and pressures satisfying the linearized equations of momentum and continuity.

Solver *pisoCentralFoam* is a combination of a central-upwind scheme [28] with the PISO algorithm.

Solver QGDFoam [29,30] is based on a system of quasi-gas dynamic equations [34–36] developed by a research team led by B.N. Chetverushkin. A quasi-gas dynamic algorithm is built on the basis of a mathematical model that generalizes the Navier-Stokes system of equations and differs from it by additional dissipative terms, having the form of second spatial derivatives with a small parameter in the form of a coefficient [36]. The principal difference of QGD (quasi-gas dynamic and quasi-hydrodynamic) systems from the Navier-Stokes system of equations is the space-time averaging for determining the main gas dynamic quantities. The presence of a controlled parameter with dissipative terms makes it possible to successfully suppress unwanted oscillations at discontinuities. The calculations used the values of this parameter in the range from 0.1 to 0.3.

To organize the comparison, the unification of calculations was performed. There are two ways in the OpenFOAM package to select the approximation variant of differential operators: directly in the solver's code or using the fvSchemes and fvSolution configuration files. To make the comparison correct, we used the same parameters, where it was possible, acting in the same way as [20-23]. The following parameters were selected in the fvSchemes file: ddtSchemes – Euler, gradSchemes – Gauss linear, divSchemes – Gauss linear, laplacianSchemes – Gauss linear corrected, interpolationSchemes – vanLeer. In the fvSolution file: solver – smoothSolver, smoother – symGaussSeidel, tolerance – 1e−09, nCorrectors – 2, nNonOrthogonalCorrectors – 1.

To estimate the deviation of the obtained numerical results from the known exact solution in the entire computational domain, analogs of the L2 norms were used

$$\partial_{L2} = \sqrt{\sum_m |y_m - y_m^{exact}|^2 V_m} \Big/ \sqrt{\sum_m |y_m^{exact}|^2 V_m}$$

and L1

$$\partial_{L1} = \sum_m |y_m - y_m^{exact}| V_m / \sum_m |y_m^{exact}| V_m.$$

Here, y_m is the pressure p, the local Mach number *Ma*, the density ρ in the cell, and V_m is the volume of the cell. All calculations were carried out with setting the following flow parameters: flow angle β = 6 °, Mach number $M_\infty = 2$, pressure $P_\infty = 101325$ Pa, temperature $T_\infty = 300$ K.

4 CALCULATION RESULTS

Calculations for all solvers allowed us to obtain a well-known flow pattern for the simulated oblique shock problem. A typical flow pattern is shown in Fig. 2 as a pressure distribution in the computational domain. The presented pressure distribution was obtained using *rhoCentralFoam* solver. The destruction of the solution was not observed for any of the solvers, which testifies to the high stabilizing properties of all solvers participating in the study.

Fig. 2. Typical pressure distribution.

For all solvers, comparisons were made with the known exact solution [2]. The results are presented in tables 1 and 2 for the norms L1 and L2, respectively. The bold font indicates the minimum values. Further, in the tables for solvers, the abbreviations are used: rCF (*rhoCentralFoam*), pCF (*pisoCentralFoam*), sF (*sonicFoam*), QGDF (*QGDFoam*). The deviations from the exact solution over the entire computational domain were calculated for the local Mach number *Ma*, pressure *p* and density ρ.

The upper row of both tables shows the value of the parameter α given for the *QGDFoam* solver, which allows adjusting the additional artificial viscosity.

	rCF	pCF	sF	QGDF, α=0.1	QGDF, α=0.15	QGDF, α=0.2	QGDF, α=0.3
Ma	**0.000592**	0.000768	0.001014	0.000646	0.000668	0.000757	0.001005
p	**0.001755**	0.001902	0.003182	0.002245	0.002203	0.002406	0.003061
ρ	**0.001350**	0.001480	0.002211	0.001549	0.001532	0.001677	0.002131

Table 1. Norm L1, M=2, β = 6°

	rCF	pCF	sF	QGDF, α=0.1	QGDF, α=0.15	QGDF, α=0.2	QGDF, α=0.3
Ma	0.004231	0.004572	0.005504	**0.004086**	0.004318	0.004699	0.005500
p	**0.013287**	0.013744	0.017505	0.014393	0.014734	0.015647	0.017753
ρ	**0.009331**	0.009633	0.012146	0.009940	0.010222	0.010860	0.012305

Table 2. Norm L2, M=2, β = 6°

The results in the tables show that the smallest deviation from the exact solution for the flow field in almost all cases is provided by the solver *rhoCentralFoam*. It can also be noted that for the *QGDFoam* solver, decreasing the α parameter significantly reduces the error. When evaluated in the L2 norm for the Mach number, the result of the QGDFoam solver provides the smallest deviation from the exact solution.

We now turn from general integral estimates to a more careful consideration of the behavior of gas-dynamic functions in the vicinity of the shock wave. Fig. 3,4,5 show the results for all solvers in the form of a density, pressure and a local Mach number distribution along the horizontal line AA_1, crossing the computational domain at a distance from the lower boundary equal to y = 0.15 (Fig 1). The exact solution is indicated by a dotted line. All solvers are indicated by the colors shown in the corresponding table in the figures.

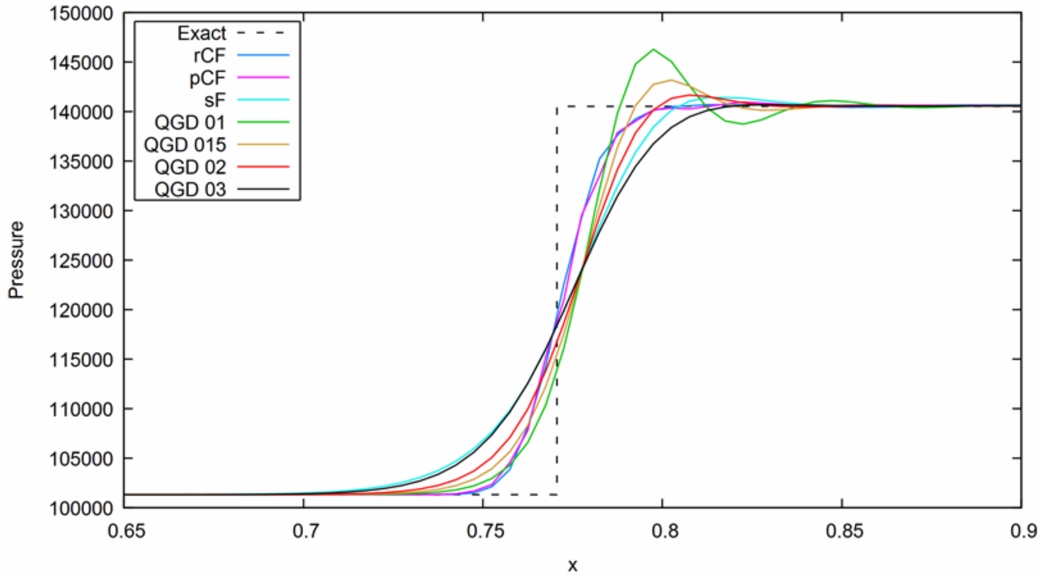

Fig.3. The distribution of pressure in the vicinity of the shock wave.

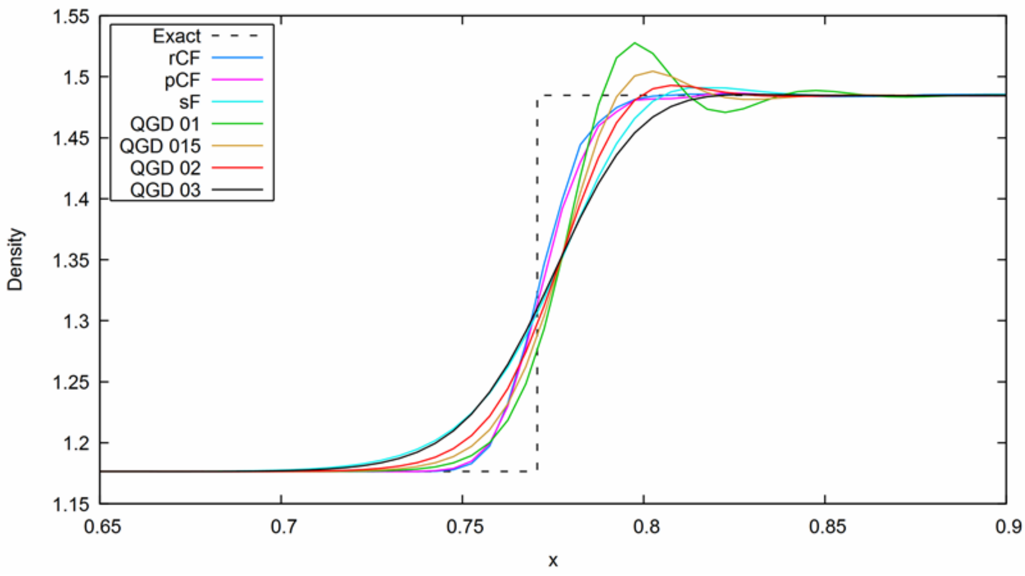

Fig.4. The distribution of density in the vicinity of the shock wave.

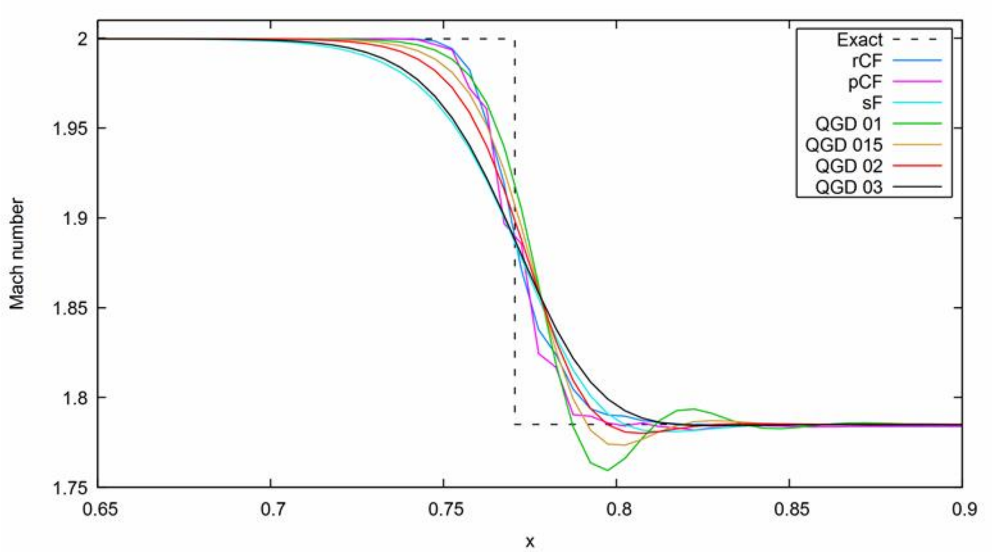

Fig.5. The distribution of the local Mach number in the vicinity of the shock wave.

The pattern of pressure distribution in the lower part of the shock and in the upper part is shown in close-ups in Figures 6 and 7, respectively.

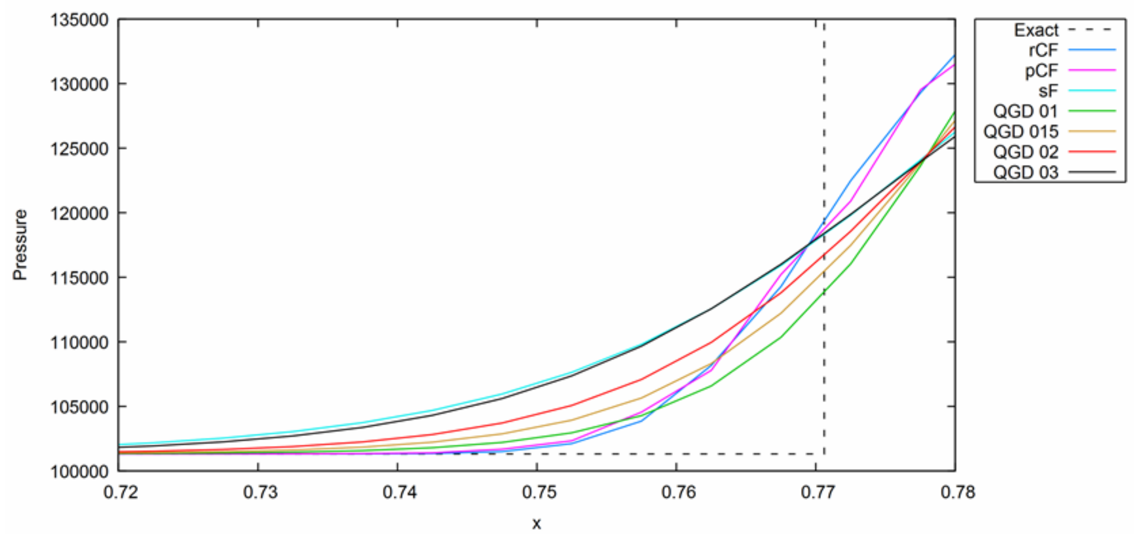

Fig.6. Pressure distribution in the vicinity of the lower part of the shock wave.

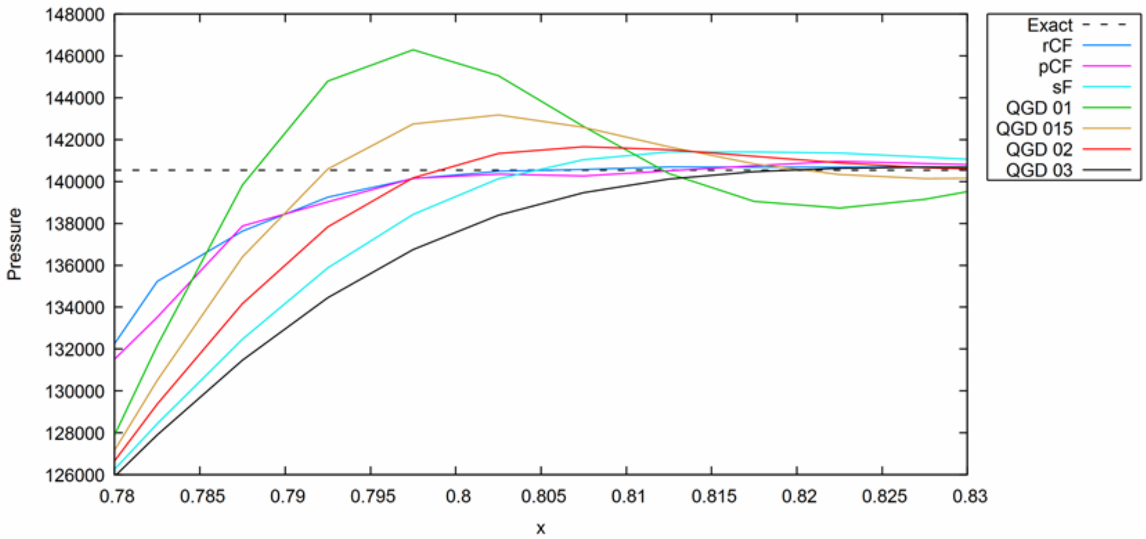

Fig.7. Pressure distribution in the vicinity of the upper part of the shock wave.

The presented figures make it possible to judge the degree of spreading of the shock for all the solvers considered in comparison. The result closest to the exact solution is provided by the *rhoCentralFoam* solver. For solver *QGDFoam*, the effect of variation of the parameter α is very clearly represented. A decrease in the parameter α brings the calculated results closer to the exact solution; however, oscillations that appear are noticeable in the upper part of the shock wave. This confirms the well-known fact that often the general assessment of accuracy in the norm of monotone schemes that provide smooth solutions shows worse results than for less monotonic schemes with oscillations. In the case of the *QGDFoam* solver, the user of the solver has the opportunity to choose either to obtain a smooth solution, or to improve the estimate at the norm.

5 CONCLUSIONS

A comparative evaluation of solution accuracy for the four OpenFOAM solvers has been made. As a test problem, we used the classical two-dimensional oblique shock problem caused by the fall of a supersonic flow of an inviscid compressible gas on a flat plate at an angle. The calculations were performed for fixed values of the parameters of the incident flow and angle of attack. Comparison of results with the exact solution was carried out over the entire field of calculated data using analogs of the norms L1 and L2.

The results obtained showed that in almost all cases the solver *rhoCentralFoam* provides the smallest deviation from the exact solution. For solver *QGDFoam*, reducing the parameter that controls the artificial viscosity can significantly reduce the deviation from the exact solution, but at the same time, oscillations appear in the upper part of the shock wave that do not destroy the solution.

The results of the comparative evaluation can be useful both for users of the OpenFoam software package and for developers of the software content of this package.

In the future, the authors consider the expansion of a comparative estimate based on the construction of a generalized computational experiment by varying the Mach number and angle of attack for the oblique shock wave problem considered. It is also planned to solve an optimization problem for the *QGDFoam* solver in order to find the optimal control of the dissipative properties of the solver on strong discontinuities.

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REFERENCES

[1] V.M. Paskonov, V.I. Polezhaev, L.A. Chudov, *Chislennoe modelirovanie processov teplo- i massoobmena*, M.: Nauka (1993).
[2] E.N. Bondarev, V.T. Dubasov, Y.A. Ryzhov et al., *Aerigidromeckanika,* M.: Mashinostroenie (1993).
[3] A.G. Kulikovsky, N.V. Pogorelov, A.Yu. Semenov, *Matematicheskie voprosy chislennogo resheniya giperbolicheskih uravneniy*, M.: Phizmatlit (2001)..
[4] A.A. Samarsky, Yu.P.Popov, *Raznosnye shemy gazovoy dinamiki*, M.: Nauka (1981).
[5] A. Harten, “High resolution schemes for hyperbolic conservation laws”, *Journal of Computational Physics*, **49,** 357-393 (1983).
[6] S. Osher, “Riemann solvers, the entropy condition, and difference approximation”, *SIAM J. Numer. Anal.*, **21** (2), 217-235 (1984).
[7] A. Harten, “ENO schemes with subcell resolution”, *Journal of Computational Physics*, **83**, 148-184 (1989).
[8] X-D. Liu, S. Osher, T. Chan, “Weighted essentially non-oscillatory schemes”, *Journal of Computational Physics*, **115**, 200-212 (1994).
[9] Ph. Colella, P.R.Woodward, “The piecewise parabolic method (PPM) for gas-dynamical simulations”, *Journal of Computational Physics*, **54** (1), 174-201 (1984).
[10] S.K. Godunov, A.V. Zabrodin, M.Ya. Ivanov et al., *Chislennoe reshenie mnogomernyh zadach gazovoy dinamiki*, M.: Nauka (1976).
[11] A. V. Zabrodin, G. P. Prokopov, V.A. Cherkashin, “Self-adapted algorithms of gas dynamics”, *Lect. Notes Phys.*, **90**, 587-593 (1979).
[12] A. V. Zabrodin, “O problemah chislennogo modelirovaniya gazodinamicheskih techeniy co

slozhnoy strukturoy", *Konstruirovanie algoritmov b reshenie zadach matematicheskoy phiziki*, M.: IPMatem. AN SSSR (1987).
[13] S. B. Bazarov, "Digital Image Processing for Visualization of Numerical Results in Fluid Dynamics", *Application of Scientific Visualization in Applied Problems*, *M: MGU*, 39–42 (2000).
[14] N. A. Dar'in, V. I. Mazhukin, A. A. Samarskii, "A finite-difference method for solving the equations of gas dynamics using adaptive grids which are dynamically connected with the solution", *U.S.S.R. Comput. Math. Math. Phys.*, **28** (4), 164–174 (1988).
[15] V.I. Mazhukin, A.A. Samarskii, M.M. Chuiko, "Metod dinamicheskoi` adaptatcii dlia chislennogo resheniia nestatcionarnykh mnogomernykh zadach Stefana", *Doclady RAN*, **368**(3), 307 – 310 (1999).
[16] V. Mazhukin, M. Chuiko. "Solution of the Multi-interface Stefan Problem by the Method of Dynamic Adaptation", *Computational Methods in Applied Mathematics*, **2**(3), 283–294 (2002). DOI: https://doi.org/10.2478/cmam-2002-0017.
[17] P.V. Breslavskii, V.I Mazhukin. "Dynamic adaptation method in gasdynamic simulations with nonlinear heat conduction", *Comput. Math. Math. Phys.,* **48**(11), 2102-2115 (2008). DOI: 10.1134/S0965542508110158).
[18] P. V. Breslavskii, V.I Mazhukin. "Dynamically adapted grids for interacting discontinuous solutions", *Comput. Math. Math. Phys.,* **47**(4), 687-706 (2007), DOI: 10.1134/S0965542507040124
[19] OpenFOAM, website: http://www.openfoam.org (accessed July 7, 2019)
[20] A.E. Bondarev, A.E. Kuvshinnikov, "Comparative study of the accuracy for OpenFOAM solvers", *Proceedings of Ivannikov ISPRAS Open Conference*. *IEEE Xplore*, 132-136 (2017). doi: 10.1109/ISPRAS.2017.00028
[21] A. E. Bondarev, A. E. Kuvshinnikov, "Analysis of the Accuracy of OpenFOAM Solvers for the Problem of Supersonic Flow Around a Cone", *Lecture Notes in Computer Science*, **10862**, 221–230 (2018). doi:10.1007/978-3-319-93713-7_18
[22] A. E. Bondarev, A. E. Kuvshinnikov, "Comparative Estimation of QGDFoam Solver Accuracy for Inviscid Flow Around a Cone", P*roceedings of the 2018 Ivannikov ISPRAS Open Conference*, *IEEE*, 82-87 (2018). doi: 10.1109/ISPRAS.2018.00019
[23] A.K. Alekseev, A.E. Bondarev, A.E. Kuvshinnikov, "Verification on the Ensemble of Independent Numerical Solutions", *Lecture Notes in Computer Science*, **11540**, 315–324 (2019). doi: 10.1007/978-3-030-22750-0_25
[24] A.E. Bondarev, V.A. Galaktionov, L.Z. Shapiro, "Postroenie I vizualniy analiz parallelnyh resheniy dlya optimizacionnyh I parametricheskih issledovaniy v vychislitelnoy gazovoy dinamike", *Proceedings of 26-th International conference of computer graphics and vision*, 211-215 (2016).
[25] A.E. Bondarev, "On the Construction of the Generalized Numerical Experiment in Fluid Dynamics", *Mathematica Montisnigri*, **42**, 52-64 (2018).
[26] A.E. Bondarev, "On visualization problems in a generalized computational experiment", *Scientific Visualization*, **11** (2), 156-162 (2019). doi: 10.26583/sv.11.2.12
[27] A.E. Bondarev, "On the Estimation of the Accuracy of Numerical Solutions in CFD Problems", *Lecture Notes in Computer Science*, **11540**, 325–333 (2019). doi: 10.1007/978-3-030-22750-0_26
[28] M. Kraposhin, A. Bovtrikova, S. Strijhak, "Adaptation of Kurganov-Tadmor numerical scheme for applying in combination with the PISO method in numerical simulation of flows in a wide range of Mach numbers", *Procedia Computer Science*, **66**, 43–52 (2015). doi:10.1016/j.procs.2015.11.007.
[29] M. V. Kraposhin, D. A. Ryazanov, E. V. Smimova, T. G. Elizarova, M. A. Istomina, "Development of OpenFOAM Solver for Compressible Viscous Flows Simulation Using Quasi-Gas Dynamic Equations", *Ivannikov ISPRAS Open Conference*, 117-123 (2017).

[30] M. A. Istomina, "About realization of one-dimensional quasi-gas dynamic algorithm in the open program OpenFOAM complex", Preprint IPM No. 1 (Moscow: KIAM), (2018).
[31] A. Kurganov, E. Tadmor, "New high-resolution central schemes for nonlinear conservation laws and convection-diffusion equations", *J. Comput. Phys.*, **160**, 241–282 (2000), doi:10.1006/jcph.2000.6459.
[32] C. J. Greenshields, H. G. Wellerr, L. Gasparini, J. M. Reese, "Implementation of semi-discrete, non-staggered central schemes in a colocated, polyhedral, finite volume framework, for high-speed viscous flows", *Int. J. Numer. Meth. Fluids*, **63** (1), 1–21 (2010), doi:10.1002/fld.2069.
[33] R.Issa, "Solution of the implicit discretized fluid flow equations by operator splitting", *J. Comput. Phys.*, **62** (1), 40–65 (1986). doi:10.1016/0021-9991(86)90099-9
[34] B. N. Chetverushkin, T. N. Elizarova "Kinetic algorithms for calculating gas dynamic flows," *U.S.S.R. Comput. Math. Math. Phys.*, **5** (5), 164–169 (1985).
[35] B. N. Chetverushkin, *Kinetic schemes and Quasi-Gas Dynamic system of equations*, CIMNE, Barcelona (2008).
[36] T. G. Elizarova, *Quasi-Gas Dynamic Equations*, Springer (2009).

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