THE ELECTROMAGNETIC AND THERMOMECHANICAL EFFECTS OF ELECTRON BEAM ON THE SOLID BARRIER

K.K. INOZEMTSEVA, M.B. MARKOV, F.N. VORONIN

Keldysh Institute of Applied Mathematics
Moscow, Russia
e-mail: m_b_markov@mail.ru

Summary: Substance behavior under high-intensive radiative exposure is considered. The thermomechanical and electromagnetic processes, which high-current electron beam initiates in a solid barrier, are modeled in their relationship. In addition to electron transport equation mathematical model includes Maxwell equations with convective current and Euler equations with the Lorentz force and Joule heating. It allows to investigate the complex radiation effect of electron beam. The expressions for the Lorentz force density dealing with ionized substance and Joule heating in substance due to the electromagnetic field are constructed. Conservative finite difference approximation is applied to the description of the electromagnetic field impact on ionized substance. The preliminary results of the thermomechanical and electromagnetic effects interaction are represented.

1. INTRODUCTION

Investigation of substance properties in extreme conditions is an actual problem. The results are important for new materials design, protection problems, etc. Laser radiation and electron beams create the extreme conditions in laboratory experiments. Lasers are used for surface excitation of investigated barrier while electron beams are applicable when volumetric energy release is needed [1].

The experiments with powerful sources of radiation are rather expensive. Measuring equipment cannot provide full quantitative data for detailed description of the interaction between radiation and substance. Therefore, mathematical modelling becomes an effective support for experimental investigation [2-6].

Radiative, thermomechanical and electromagnetic effects accompany the interaction between an electron beam and a solid barrier. All of them arise from electron transport and scattering. The beam interacts with the barrier through elastic scattering, bremsstrahlung, impact ionization and excitation. [7-9]. It is impact ionization that is the main channel of energy transfer from electrons to the barrier. The differential ionization cross section is inversely proportional to the square of the energy transfer magnitude. So, a lot of low energy secondary electrons cross to so-called ionization spectrum [10-11]. They degrade to equilibrium with energy transfer to scattering medium [12]. Excess charge carriers cause radiation-induced conductivity, energy transfer leads to medium heating [13]. Heating causes substance evaporation, pressure increase and shock wave generation [14]. Nonuniform thermal and mechanical fields arouse thermomechanical effects: deformation, melting, evaporation, etc [14].

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Electron movement creates current density which generates the electromagnetic field. The nonuniform current density causes bulk electric charge [15]. Electromagnetic field is a reason for electromagnetic effects: electric current of conductivity, electric breakdown, etc.

Such collision process as bremsstrahlung should be considered separately [9]. Most of the existing high-current accelerators generate electrons with the energy less than 10 MeV. Corresponding bremsstrahlung photons receive energy near 2 MeV [6]. The free path of such photon exceeds the electron's one by two orders of magnitude. As a result, the size of ionized region exceeds the electron free path sufficiently. Bremsstrahlung photons experience Compton and coherent scattering, photo absorption, produce electron-positron pairs [16]. Consequently, bremsstrahlung photons turn into the flux of charged particles conversely.

Radiative, thermomechanical and electromagnetic fields affect each other. Density redistribution during the dumping of the mechanic stresses changes the scattering properties of substance. Ionization during radiative heating enhances the conductivity, which reduces the electric field. The Electric field and the bulk charge create the ponderomotive force. It moves substance along with the pressure gradient. Joule heating of conductive substance in the electric field causes additional energy release.

Keldysh Institute researchers have developed a program package REMP (Radiation and ElectroMagnetic Pulse) for mathematical modelling of physical effects which accompany the interaction of electrons and photons with complex technical objects. It includes computational modules and the user interface, connected by unified data communication protocol. The classic transport equation describes free electrons scattering. The Monte-Carlo method provides means for the direct modelling of particle collisions. The classic kinetic equations describe particle propagation in vacuum and gases. The Maxwell equations describe the electromagnetic field evolution. The mentioned equations form the arsenal to consider the transport processes, the electromagnetic field, both external and self-consistent, and their interaction. The computational modules are connected through a program script with the hydrodynamic code MARPLE-3D [17]. It enables to consider the dynamics of substance exposed to radiation. The quantum kinetic equations for conductivity electrons and holes of valence band describe the radiation-induced conductivity of semiconductors. The free electron energy release serves as a source in it. The collision integral in the quantum equation describes scattering on phonons of crystal lattice. The statistical particles method solves numerically both the quantum and the classic kinetic equations. The method combines stochastic scattering simulation with the equations of motion in electromagnetic field which are solved between collisions [18]. The program package is operated at the heterogeneous cluster K-100.

This article is devoted to the script development for simulation of interacting thermomechanical and electromagnetic fields of radiation genesis.

2. PROBLEM STATEMENT

The following kinetic equation describes fast electrons transport and scattering:

$$\frac{\partial f_e}{\partial t} + \mathrm{div}(\mathbf{x} f_e) + e\,\mathrm{div}_p\left[\left(\mathbf{E} + \frac{1}{c}\left[\mathbf{x},\mathbf{B}\right]\right) f_e\right] + \sigma_e^t \upsilon f_e = c\int d\mathbf{p}'\sigma_{ph-e}(\mathbf{p},\mathbf{p}') f_{ph}(\mathbf{p}') + \\ + \int d\mathbf{p}'\sigma_{e\text{-}e}(\mathbf{p},\mathbf{p}')\upsilon' f_e(\mathbf{p}') + \int d\mathbf{p}'\sigma_{p-e}(\mathbf{p},\mathbf{p}')\upsilon' f_p(\mathbf{p}') + Q_e, \tag{1}$$

where $f_e = f_e(t,\mathbf{r},\mathbf{p})$, $f_p = f_p(t,\mathbf{r},\mathbf{p})$, $f_{ph} = f_{ph}(t,\mathbf{r},\mathbf{p})$ are the distribution functions of electrons, positrons and photons respectively in the phase space of coordinates $\mathbf{r}$ and momentum $\mathbf{p}$, $\mathbf{x}$ is particle velocity, c is the speed of light, e is an electron charge, $\mathbf{E} = \mathbf{E}(t,\mathbf{r})$, $\mathbf{B} = \mathbf{B}(t,\mathbf{r})$ are the electric field strength and the magnetic induction, $\mathbf{p'}$ is the particle momentum before collision, div_p is the divergence operator in momentum space, σ_e^t is the total cross section of electron adsorption, σ_{ph-e}, σ_{e-e}, σ_{p-e} are the differential cross sections of photon, electron, and positron scattering with electron generation, $Q_e = Q_e(t,\mathbf{r},\mathbf{p})$ is the electron source.

The following kinetic equation describes positrons and photons which are generated in cascade processes:

$$\frac{\partial f_p}{\partial t} + \mathrm{div}(\mathbf{x} f_p) - e\,\mathrm{div}_p\left[\left(\mathbf{E} + \frac{1}{c}[\mathbf{x},\mathbf{B}]\right) f_p\right] + \sigma_p^t \upsilon f_p = c\int d\mathbf{p'}\sigma_{ph-p}(\mathbf{p},\mathbf{p'}) f_{ph}(\mathbf{p'}) + \int d\mathbf{p'}\sigma_{p-p}(\mathbf{p},\mathbf{p'})\upsilon' f_p(\mathbf{p'}), \tag{2}$$

$$\frac{\partial f_{ph}}{\partial t} + c\,\mathrm{div}(\mathbf{\Omega} f_e) + c\sigma_{ph}^t f_e = c\int d\mathbf{p'}\sigma_{ph-ph}(\mathbf{p},\mathbf{p'}) f_{ph}(\mathbf{p'}) + \int d\mathbf{p'}\sigma_{e\text{-}ph}(\mathbf{p},\mathbf{p'})\upsilon' f_e(\mathbf{p'}) + \int d\mathbf{p'}\sigma_{p-ph}(\mathbf{p},\mathbf{p'})\upsilon' f_p(\mathbf{p'}), \tag{3}$$

where $\mathbf{\Omega}$ is the unit vector of photon velocity, σ_{ph}^t is the total cross section of photon adsorption, σ_p^t is the total cross section of positron adsorption, σ_{ph-p}, σ_{p-p} are the differential cross sections of photon and positron scattering with positron generation, σ_{ph-ph}, σ_{e-ph}, σ_{p-ph} are the differential cross sections of photon, electron, and positron scattering with photon generation.

The equations (1), (2), (3) are considered in the space of finitary generalized functions [19]. The details of the methods of equation solving are given in [18, 20, 21].

Let's consider the following mathematical constructions:

$$Q_\varepsilon^e(t,\mathbf{r}) \equiv \int d\mathbf{p}\int d\mathbf{r}'\varepsilon(p) W\left(|\mathbf{r}-\mathbf{r}'|,\Delta\right)\left[\sigma_e^t \upsilon f_e(t,\mathbf{r}',\mathbf{p}) - \int d\mathbf{p'}\sigma_e(\mathbf{p},\mathbf{p'})\upsilon' f_e(t,\mathbf{r}',\mathbf{p'})\right], \tag{4}$$

$$Q_\varepsilon^p(t,\mathbf{r}) \equiv \int d\mathbf{p}\int d\mathbf{r}'\varepsilon(p) W\left(|\mathbf{r}-\mathbf{r}'|,\Delta\right)\left[\sigma_p^t \upsilon f_p(t,\mathbf{r}',\mathbf{p}) - \int d\mathbf{p'}\sigma_p(\mathbf{p},\mathbf{p'})\upsilon' f_p(t,\mathbf{r}',\mathbf{p'})\right], \tag{5}$$

$$Q_\varepsilon^{ph}(t,\mathbf{r}) \equiv \int d\mathbf{p}\int d\mathbf{r}'\varepsilon(p) W\left(|\mathbf{r}-\mathbf{r}'|,\Delta\right)\left[\sigma_{ph}^t \upsilon f_{ph}(t,\mathbf{r}',\mathbf{p}) - \int d\mathbf{p'}\sigma_{ph}(\mathbf{p},\mathbf{p'})\upsilon' f_{ph}(t,\mathbf{r}',\mathbf{p'})\right], \tag{6}$$

where $\varepsilon(p)$ – energy of particle, infinitely differentiable function $W\left(|\mathbf{r}-\mathbf{r}'|,\Delta\right)$, $\mathbf{r}' \in \mathbb{R}^3_{\mathbf{r}}$, $\Delta > 0$ satisfies conditions $\int_{\mathbb{R}^3_{\mathbf{r}}} W\left(|\mathbf{r}-\mathbf{r}'|,\Delta\right) d\mathbf{r}' = 1$, $\lim\limits_{\Delta\to 0}\int_{\mathbb{R}^3_{\mathbf{r}}} d\mathbf{r} W\left(|\mathbf{r}-\mathbf{r}'|,\Delta\right)\varphi(\mathbf{r},\mathbf{p}) = \varphi(\mathbf{r}',\mathbf{p})$ for every infinitely differentiable function from the pivot space. The following relations connect the differential scattering cross sections in (4), (5), (6) with summands of collision integrals in the equations (1), (2), (3):

$$\sigma_e(\mathbf{p},\mathbf{p}') = \sigma_{e-e}(\mathbf{p},\mathbf{p}') + \sigma_{e-ph}(\mathbf{p},\mathbf{p}'),$$
$$\sigma_p(\mathbf{p},\mathbf{p}') = \sigma_{p-e}(\mathbf{p},\mathbf{p}') + \sigma_{p-ph}(\mathbf{p},\mathbf{p}') + \sigma_{p-p}(\mathbf{p},\mathbf{p}'),$$
$$\sigma_{ph}(\mathbf{p},\mathbf{p}') = \sigma_{ph-e}(\mathbf{p},\mathbf{p}') + \sigma_{ph-ph}(\mathbf{p},\mathbf{p}') + \sigma_{ph-p}(\mathbf{p},\mathbf{p}').$$

The expressions (4), (5), (6) determine the energy power density which electrons, positrons and photons transmit to scattering medium. The total energy release is:

$$Q^{\varepsilon} = Q_{\varepsilon}^{e} + Q_{\varepsilon}^{p} + Q_{\varepsilon}^{ph}. \tag{7}$$

Let's consider constructions:

$$\mathbf{j}_{ext}(t,\mathbf{r}) \equiv e\int d\mathbf{p}\int d\mathbf{r}'\mathbf{x}W\left(|\mathbf{r}-\mathbf{r}'|,\Delta\right)\left[f_e(t,\mathbf{r}',\mathbf{p}) - f_p(t,\mathbf{r}',\mathbf{p})\right], \tag{8}$$

$$q_{ext}(t,\mathbf{r}) \equiv e\int d\mathbf{p}\int d\mathbf{r}'W\left(|\mathbf{r}-\mathbf{r}'|,\Delta\right)\left[f_e(t,\mathbf{r}',\mathbf{p}) - f_p(t,\mathbf{r}',\mathbf{p})\right], \tag{9}$$

$$\begin{aligned}\mathbf{Q}^{p}(t,\mathbf{r}) \equiv e\int d\mathbf{p}\int d\mathbf{r}'\mathbf{p}W\left(|\mathbf{r}-\mathbf{r}'|,\Delta\right)\Big[&\sigma_e^t\upsilon f_e(t,\mathbf{r}',\mathbf{p}) - \int d\mathbf{p}'\sigma_e(\mathbf{p},\mathbf{p}')\upsilon' f_e(t,\mathbf{r}',\mathbf{p}') + \\ &+\sigma_p^t\upsilon f_p(t,\mathbf{r}',\mathbf{p}) - \int d\mathbf{p}'\sigma_p(\mathbf{p},\mathbf{p}')\upsilon' f_p(t,\mathbf{r}',\mathbf{p}') + \\ &+\sigma_{ph}^t\upsilon f_{ph}(t,\mathbf{r}',\mathbf{p}) - \int d\mathbf{p}'\sigma_{ph}(\mathbf{p},\mathbf{p}')\upsilon' f_{ph}(t,\mathbf{r}',\mathbf{p}')\Big],\end{aligned} \tag{10}$$

$$\begin{aligned}Q^{\rho}(t,\mathbf{r}) \equiv \int d\mathbf{p}\int d\mathbf{r}'mW\left(|\mathbf{r}-\mathbf{r}'|,\Delta\right)\Big[&\sigma_e^t\upsilon f_e(t,\mathbf{r}',\mathbf{p}) - \int d\mathbf{p}'\sigma_e(\mathbf{p},\mathbf{p}')\upsilon' f_e(t,\mathbf{r}',\mathbf{p}') + \\ &+\sigma_p^t\upsilon f_p(t,\mathbf{r}',\mathbf{p}) - \int d\mathbf{p}'\sigma_p(\mathbf{p},\mathbf{p}')\upsilon' f_p(t,\mathbf{r}',\mathbf{p}')\Big],\end{aligned} \tag{11}$$

where m is electron mass and the vector $\mathbf{Q}^p$ has components Q_i^p, $i=1,2,3$.

The formulae (8), (9) express charge and external electric current density. The formulae (10), (11) express momentum and mass transfer to scattering medium. The represented definitions of the model attributes provide model applicability for heavy particle fluxes.

Energy transfer to the barrier causes heating, melting or evaporation. Every process takes place in actual cases. There are regions of barrier where energy release exceeds, matches or is less than the heat of sublimation. Different equations describe this situations. But it is impossible to extract domains of applicability for this equations a priori. We use ideal hydrodynamic Euler equations [22] for all cases. Complex tabular equations of state describe detailed properties of substance [23]. The following Euler equations are classic:

$$\frac{\partial\rho}{\partial t} + \mathrm{div}\rho\mathbf{v} = Q^{\rho}, \tag{12}$$

$$\frac{\partial}{\partial t}\rho v_i + \frac{\partial}{\partial x_k}\left(\rho v_i v_k + p\delta_{ik}\right) = Q_i^p, \tag{13}$$

$$\frac{\partial}{\partial t}\left(\rho\left(\frac{v^2}{2}+u\right)\right) + \mathrm{div}\left(\mathbf{v}\rho\left(\frac{v^2}{2}+u\right) + \mathbf{v}p\right) = Q^{\varepsilon}, \tag{14}$$

where ρ is the density of barrier substance, p is pressure, T is temperature, u is internal energy, $\mathbf{v}$ is the specific velocity of barrier substance with components v_i, $i=1,2,3$. The Summation convention is applied.

Electrons interaction with the barrier is accompanied by generation of electric charge $q_{ext}(t,\mathbf{r})$ and current $\mathbf{j}_{ext}(t,\mathbf{r})$ densities. Electric current, in its turn, generates electromagnetic field. It creates ponderomotive force which sets ionized substance in motion. So, balances of momentum (13) and energy (14) in ionized substance should be transformed in the corresponding relation for the system including substance and electromagnetic field [24]. The extrinsic current work $\mathbf{j}_{ext}\mathbf{E}$ should be subtracted from the external energy gain. A new source of momentum emerges is the Lorentz force $-q_{ext}\mathbf{E}-\left[\mathbf{j}_{ext}\times\mathbf{B}\right]/c$ acting on beam electrons. The following Euler equations describe ionized substance and the field:

$$\frac{\partial\rho}{\partial t}+\mathrm{div}\rho\mathbf{v}=Q^{m}, \tag{15}$$

$$\frac{\partial}{\partial t}\left(\rho v_i+g_i\right)=-\frac{\partial}{\partial x_k}\left(\rho v_i v_k+\left(p-p_{str}\right)\delta_{ik}-T_{ik}\right)+Q_i^{p}-\left(q_{ext}E_i+\frac{1}{c}\left[\mathbf{j}_{ext}\times\mathbf{B}\right]_i\right), \tag{16}$$

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\rho\left(\frac{v^2}{2}+u\right)+\frac{E^2}{8\pi}+\frac{B^2}{8\pi}-\mathbf{M'B}+\frac{\mathbf{v}}{c}\left[\mathbf{M'}\times\mathbf{E'}\right]\right)=Q^{\varepsilon}-\mathbf{j}_{ext}\mathbf{E}-\\&-\mathrm{div}\left(\mathbf{v}\rho\left(\frac{v^2}{2}+u\right)+\mathbf{S}+\mathbf{v}\left(p-p_{str}-\mathbf{P'}\left(\mathbf{E}-\frac{\mathbf{E'}}{2}\right)-\mathbf{M'}\left(\mathbf{B}-\frac{\mathbf{B'}}{2}\right)\right)\right),\end{aligned} \tag{17}$$

where for isotropic substance

$$T_{ik}=\frac{1}{4\pi}\left(E'_i D'_k+H'_i B'_k-\delta_{ik}\left(\frac{1}{2}\mathbf{E'D'}+\frac{1}{2}\mathbf{H'B'}\right)-\frac{1}{c}\left(\left[\mathbf{E'}\times\mathbf{H'}\right]_i v_k+\left[\mathbf{E'}\times\mathbf{H'}\right]_k v_i\right)\right), \tag{18}$$

vectors $\mathbf{E}$, $\mathbf{P}$, $\mathbf{D}$, $\mathbf{H}$, $\mathbf{M}$, $\mathbf{B}$ with components E_i, P_i, D_i, H_i, M_i, B_i represent the electric field strength, the polarization, the electric displacement, the magnetic field strength, the magnetization and the magnetic induction, respectively, $\mathbf{S}$ is the Poynting vector, g_i is the electromagnetic field momentum density, p_{str} is the striction pressure.

$$\mathbf{g}=\frac{1}{4\pi c}\left[\mathbf{E}\times\mathbf{H}\right], \tag{19}$$

$$\mathbf{S}=\frac{c}{4\pi}\left[\mathbf{E}\times\mathbf{H}\right]. \tag{20}$$

Vectors $\mathbf{E'}$, $\mathbf{P'}$, $\mathbf{D'}$, $\mathbf{H'}$, $\mathbf{M'}$, $\mathbf{B'}$ with components E'_i, P'_i, D'_i, H'_i, M'_i, B'_i represent the electric field strength, the polarization, the electric displacement, the magnetic field strength, the magnetization and the magnetic induction in the substance's rest frame.

$$\mathbf{D'}=\mathbf{D}+\left[\frac{\mathbf{v}}{c}\times\mathbf{H'}\right], \tag{21}$$

$$\mathbf{E'}=\mathbf{E}+\left[\frac{\mathbf{v}}{c}\times\mathbf{B'}\right], \tag{22}$$

$$\mathbf{P'} = \mathbf{P} - \left[\frac{\mathbf{v}}{c} \times \mathbf{M'}\right], \tag{23}$$

$$\mathbf{H'} = \mathbf{3} - \left[\frac{\mathbf{v}}{c} \times \mathbf{D'}\right], \tag{24}$$

$$\mathbf{B'} = \mathbf{B} - \left[\frac{\mathbf{v}}{c} \times \mathbf{E'}\right], \tag{25}$$

$$\mathbf{M'} = \mathbf{M} + \left[\frac{\mathbf{v}}{c} \times \mathbf{P'}\right], \tag{26}$$

We propose the following connections between $\mathbf{E'}$ and $\mathbf{D'}$, $\mathbf{B'}$ and $\mathbf{H'}$:

$$\mathbf{D'} = \varepsilon \mathbf{E'}, \tag{27}$$

$$\mathbf{B'} = \mu \mathbf{H'}, \tag{28}$$

where the dielectric permittivity ε and permeability μ depend on thermodynamic parameters only. This dependence causes the striction pressure p_{str} [24]:

$$p_{str} = \frac{E'^2}{8\pi} \rho \frac{\partial \varepsilon}{\partial \rho} + \frac{H'^2}{8\pi} \rho \frac{\partial \mu}{\partial \rho}. \tag{29}$$

Thermodynamic relations [24] determine the following consequence for dielectrics. Specific internal energy can be represented as a sum of components. One of them u_0 is independent from electric field:

$$u = u_0 + \frac{\mathbf{E'P'}}{2\rho} + \frac{\mathbf{B'M'}}{2\rho} + \frac{E'^2}{8\pi\rho} T \frac{\partial \varepsilon}{\partial T} + \frac{H'^2}{8\pi\rho} T \frac{\partial \mu}{\partial T}. \tag{30}$$

Let's consider Maxwell equations [25] for electromagnetic field in continuous media:

$$\mathrm{rot}\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\left(\mathbf{j}_{ext} + \mathbf{j}^* + q\mathbf{v}\right), \tag{31}$$

$$\mathrm{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{32}$$

$$\mathrm{div}\,\mathbf{D} = 4\pi\left(q + q_{ext}\right), \tag{33}$$

where q is charge density in ionized substance, $\mathbf{j}^*$ is the current density of conductivity electrons. In the frame of Cauchy problem for equations (31-33) Coulomb law (33) is equivalent to charge continuity equation:

$$\frac{\partial\left(q + q_{ext}\right)}{\partial t} + \mathrm{div}\left(\mathbf{j}_{ext} + \mathbf{j}^* + q\mathbf{v}\right) = 0.$$

The law of electromagnetic field energy conservation follows from Maxwell equations (31-33):

$$\frac{\partial}{\partial t}\left(\frac{E^2}{8\pi}+\frac{B^2}{8\pi}\right)+\mathbf{E}\frac{\partial \mathbf{P}}{\partial t}-\mathbf{M}\frac{\partial \mathbf{B}}{\partial t}=-\mathrm{div}\mathbf{S}-\mathbf{jE}, \tag{34}$$

where $\mathbf{j}$ is the total current density:

$$\mathbf{j}=\mathbf{j}^{*}+q\mathbf{v}+\mathbf{j}_{ext}. \tag{35}$$

Let's multiply **Ошибка! Источник ссылки не найден.** by v_i and sum over index i:

$$v_i\frac{\partial}{\partial t}\left(\rho v_i+g_i\right)=v_iQ_i^p-v_i\left(q_{ext}E_i+\frac{1}{c}\left[\mathbf{j}_{ext}\times\mathbf{B}\right]_i\right)-v_i\frac{\partial}{\partial x_k}\left(\rho v_iv_k+\left(p-p_{str}\right)\delta_{ik}-T_{ik}\right). \tag{36}$$

After term rearranging in (36):

$$\begin{aligned}v^2\frac{\partial\rho}{\partial t}+\rho\frac{\partial}{\partial t}\left(\frac{v^2}{2}\right)=-v^2\frac{\partial}{\partial x_k}\rho v_k-\rho v_k\frac{\partial}{\partial x_k}\left(\frac{v^2}{2}\right)-v_i\left(q_{ext}E_i+\frac{1}{c}\left[\mathbf{j}_{ext}\times\mathbf{B}\right]_i\right)-\\-v_i\frac{\partial}{\partial x_k}\left(\rho v_iv_k+\left(p-p_{str}\right)\delta_{ik}-T_{ik}\right)+v_iQ_i^p-v_i\frac{\partial g_i}{\partial t}.\end{aligned} \tag{37}$$

The equation for kinetic energy balance follows from (37) and (15):

$$\begin{aligned}\frac{\partial}{\partial t}\left(\rho\frac{v^2}{2}\right)=-v_i\frac{\partial}{\partial x_k}\left(\rho v_iv_k+\left(p-p_{str}\right)\delta_{ik}-T_{ik}\right)-\frac{\partial}{\partial x_k}\left(v_k\rho\frac{v^2}{2}\right)-v_i\frac{\partial g_i}{\partial t}-\\-v_i\left(q_{ext}E_i+\frac{1}{c}\left[\mathbf{j}_{ext}\times\mathbf{B}\right]_i\right)+v_iQ_i^p.\end{aligned} \tag{38}$$

Relations (17), (30), (34), (38) determine the law of internal energy conservation. One can use it instead of (17):

$$\begin{aligned}\rho\frac{d}{dt}\left(u_0+T\frac{E'^2}{8\pi}\frac{\partial\varepsilon}{\partial T}+T\frac{H'^2}{8\pi}\frac{\partial\mu}{\partial T}\right)=-\left(p-p_{str}\right)\mathrm{div}\mathbf{v}+\zeta+Q^{\varepsilon}-v_iQ_i^p+\left(\mathbf{j}-\mathbf{j}_{ext}\right)\mathbf{E}+\\+v_i\frac{\partial g_i}{\partial t}-v_i\frac{\partial}{\partial x_k}T_{ik}+v_i\left(q_{ext}E_i+\frac{1}{c}\left[\mathbf{j}_{ext}\times\mathbf{B}\right]_i\right),\end{aligned} \tag{39}$$

where $\frac{d}{dt}=\frac{\partial}{\partial t}+\mathbf{v}\nabla$ is the total derivative,

$$\begin{aligned}\zeta=\frac{\partial}{\partial t}\left(\mathbf{M'B}-\frac{\mathbf{v}}{c}\left[\mathbf{M'}\times\mathbf{E'}\right]-\frac{\mathbf{E'P'}}{2}-\frac{\mathbf{B'M'}}{2}\right)+\mathrm{div}\left(\mathbf{v}\left(\mathbf{P'}\left(\mathbf{E}-\mathbf{E'}\right)+\mathbf{M'}\left(\mathbf{B}-\mathbf{B'}\right)\right)\right)+\\+\mathbf{E}\frac{\partial\mathbf{P}}{\partial t}-\mathbf{M}\frac{\partial\mathbf{B}}{\partial t}.\end{aligned} \tag{40}$$

The derivation of the equation (39) is founded on the following assumption. Thermodynamic parameters determine dielectric permittivity uniquely. It means that only locally equilibrium and reversible processes change permittivity. Substance dynamics equations (16), (17) are formulated as laws of conservation. Therefore equations validity is

connected with the applicability of the thermodynamic relations for substance under consideration (for example (30)). Equations (15), (16), (17) hold true if the relaxation times of the processes, which are connected with permittivity change, are small in comparison with the characteristic macroscopic time. The law of electromagnetic energy conservation (34) doesn't change, so the equation (39) holds true.

One can simplify (39) with the help of momentum conservation law [24]:

$$\frac{\partial}{\partial x_k} T_{ik} - \frac{\partial g_i}{\partial t} = q_{ext} E_i + \frac{1}{c}\left[\mathbf{j}_{ext} \times \mathbf{B}\right]_i + F_i \tag{41}$$

where

$$F_i = qE_i + \frac{1}{c}\left[\left(\mathbf{j} - \mathbf{j}_{ext}\right) \times \mathbf{B}\right]_i + \mathbf{P}' \frac{\partial \mathbf{E}'}{\partial x_i} + \mathbf{M}' \frac{\partial \mathbf{B}'}{\partial x_i} + \frac{1}{c} \frac{\partial \mathbf{v}}{\partial x_i} \left(\left[\mathbf{P}' \times \mathbf{B}'\right] - \left[\mathbf{M}' \times \mathbf{E}'\right]\right) + \tag{42}$$

$$+ \frac{\rho}{c} \frac{d}{dt} \left(\left[\mathbf{p}' \times \mathbf{B}'\right]_i - \left[\mathbf{m}' \times \mathbf{E}'\right]_i\right) - \frac{1}{2} \frac{\partial}{\partial x_i} \left(\mathbf{P}'\mathbf{E}' + \mathbf{M}'\mathbf{B}'\right)$$

$$\mathbf{p}' = \mathbf{P}' / \rho \tag{43}$$

$$\mathbf{m}' = \mathbf{M}' / \rho \tag{44}$$

The next relation follows from (39) and (41):

$$\rho \frac{d}{dt} \left(u_0 + T \frac{E'^2}{8\pi} \frac{\partial \varepsilon}{\partial T} + T \frac{H'^2}{8\pi} \frac{\partial \mu}{\partial T} \right) = -\left(p - p_{str}\right) \operatorname{div} \mathbf{v} + \zeta + Q^{\varepsilon} - v_i Q_i^p + \left(\mathbf{j} - \mathbf{j}_{ext}\right) \mathbf{E} - \mathbf{v}\mathbf{F} \tag{45}$$

Using the following relations

$$v_i \frac{\partial \mathbf{E}'}{\partial r_i} \mathbf{P}' = \operatorname{div}\left(\left(\mathbf{P}'\mathbf{E}'\right)\mathbf{v}\right) + \mathbf{E}' \frac{\partial \mathbf{P}'}{\partial t} - \rho \mathbf{E}' \frac{d\mathbf{p}'}{dt}, \tag{46}$$

$$v_i \frac{\partial \mathbf{B}'}{\partial r_i} \mathbf{M}' = \operatorname{div}\left(\left(\mathbf{M}'\mathbf{B}'\right)\mathbf{v}\right) + \mathbf{B}' \frac{\partial \mathbf{M}'}{\partial t} - \rho \mathbf{B}' \frac{d\mathbf{m}'}{dt}, \tag{47}$$

$$v_i \frac{\partial \mathbf{v}}{\partial r_i} \left(\left[\mathbf{P}' \times \mathbf{B}'\right] - \left[\mathbf{M}' \times \mathbf{E}'\right]\right) = \operatorname{div}\left(v^2 \left[\mathbf{P}' \times \mathbf{B}'\right] - \left[\mathbf{M}' \times \mathbf{E}'\right]\right) + \mathbf{v} \frac{\partial}{\partial t} \left(\left[\mathbf{P}' \times \mathbf{B}'\right] - \left[\mathbf{M}' \times \mathbf{E}'\right]\right) - \tag{48}$$

$$- \rho \mathbf{v} \frac{d}{dt} \left(\left[\mathbf{p}' \times \mathbf{B}'\right] - \left[\mathbf{m}' \times \mathbf{E}'\right]\right),$$

and neglecting terms of the order of $\mathbf{v}^2 / c^2$, we obtain:

$$\begin{aligned} \mathbf{v}\mathbf{F} = \operatorname{div}\left(\left(\mathbf{P}'\left(\mathbf{E} - \frac{\mathbf{E}'}{2}\right) + \mathbf{M}'\left(\mathbf{B} - \frac{\mathbf{B}'}{2}\right)\right)\mathbf{v}\right) + \mathbf{E} \frac{\partial \mathbf{P}}{\partial t} - \mathbf{M} \frac{\partial \mathbf{B}}{\partial t} + \frac{\partial}{\partial t}\left(\mathbf{M}'\mathbf{B}\right) - \\ - \frac{1}{c} \frac{\partial}{\partial t} \left(\mathbf{v}\left[\mathbf{M}' \times \mathbf{E}'\right]\right) + q\mathbf{E}\mathbf{v} + \frac{\mathbf{v}}{c}\left[\mathbf{j}^* \times \mathbf{B}\right] - \rho \mathbf{E}' \frac{d\mathbf{p}'}{dt} - \rho \mathbf{B}' \frac{d\mathbf{m}'}{dt} \end{aligned} \tag{49}$$

Substituting $\mathbf{v}\mathbf{F}$ from (49) into (39), one can obtain:

$$\rho\frac{d}{dt}\left(u_0+T\frac{\partial\varepsilon}{\partial T}\frac{E^2}{8\pi}\right)=-(p-p_{str})\mathrm{div}\mathbf{v}+Q^{\varepsilon}-\mathbf{v}\mathbf{Q}^{p}+\mathbf{j}^{*}\mathbf{E}-\frac{\mathbf{v}}{c}\left[\mathbf{j}^{*}\times\mathbf{B}\right]-$$
$$-\frac{\partial}{\partial t}\left(\frac{\mathbf{E'P'}}{2}+\frac{\mathbf{M'B'}}{2}\right)-\mathrm{div}\left(\mathbf{v}\frac{\left(\mathbf{E'P'}\right)}{2}+\mathbf{v}\frac{\left(\mathbf{M'B'}\right)}{2}\right)+\rho\mathbf{E'}\frac{d\mathbf{p'}}{dt}+\rho\mathbf{B'}\frac{d\mathbf{m'}}{dt}. \tag{50}$$

Using (27), (28), one can simplify (50):

$$\rho\frac{d}{dt}\left(u_0+T\frac{\partial\varepsilon}{\partial T}\frac{E^2}{8\pi}\right)=-(p-p_{str})\mathrm{div}\mathbf{v}+Q^{\varepsilon}-\mathbf{v}\mathbf{Q}^{p}+\mathbf{j}^{*}\mathbf{E}-\frac{\mathbf{v}}{c}\left[\mathbf{j}^{*}\times\mathbf{B}\right]+$$
$$+\frac{E'^{2}}{8\pi}\frac{d\varepsilon}{dt}+\frac{H'^{2}}{8\pi}\frac{d\mu}{dt}. \tag{51}$$

One can obtain another form of momentum conservation law for the system of substance and field instead of (16) with the help of (32):

$$\rho\frac{dv_i}{dt}=-\frac{\partial}{\partial x_i}\left(p-p_{str}\right)+qE'_i+\frac{1}{c}\left[\mathbf{j}^{*}\times\mathbf{B'}\right]_i+\frac{1}{c}\frac{\partial\mathbf{v}}{\partial x_i}\left(\left[\mathbf{P'}\times\mathbf{B'}\right]-\left[\mathbf{M'}\times\mathbf{E'}\right]\right)-$$
$$+\frac{1}{2}\left(\left(\mathbf{P'}\frac{\partial\mathbf{E'}}{\partial x_i}-\mathbf{E'}\frac{\partial\mathbf{P'}}{\partial x_i}\right)+\left(\mathbf{M'}\frac{\partial\mathbf{B'}}{\partial x_i}-\mathbf{B'}\frac{\partial\mathbf{M'}}{\partial x_i}\right)\right)+\frac{\rho}{c}\frac{d}{dt}\left(\left[\mathbf{p'}\times\mathbf{B'}\right]_i-\left[\mathbf{m'}\times\mathbf{E'}\right]_i\right). \tag{52}$$

The expression for current density of conductivity electrons completes equations (22-24):

$$\mathbf{j}^{*}=\sigma\mathbf{E'}=\sigma\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B'}\right)=\sigma\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right), \tag{53}$$

where σ is total thermal and radiative conductivity.

One can obtain the relation for thermal conductivity from the Wiedemann-Francis law. It connects electric and heat conductivity. The electron heat conductivity λ is considered in approximation [23]:

$$\lambda=4ac\theta^{3}l/3, \tag{54}$$

where $a=\pi^{2}/15c^{3}\hbar^{3}$.

$$l=\sqrt{\lambda_S{}^{2}+\lambda_H{}^{2}}\Big/\theta^{3}\,[\text{см}], \tag{55}$$

where λ_S and λ_H are the thermal conductivity coefficients for ideal nondegenerate and tightly degenerate plasma:

$$\lambda_S=1{,}17\cdot10^{-3}\frac{e_1\theta^{5/2}}{Z_{eff}L_1}, \tag{56}$$

$$\lambda_H=3{,}10\cdot10^{-4}\frac{e_2\theta\theta_F^{3/2}}{Z_{eff}L_2}. \tag{57}$$

The factors e_1 and e_2 provide correction for plasma nonideality. Atomic units are used for substance θ and Fermi θ_F temperatures. The Coulomb logarithms of electron-ion collisions can be calculated in the next approximations:

$$L_2 = 0.5 \ln\left[\left(2\pi Z_{eff}/3\right)^{2/3} \left(1.5 + 3/\Gamma\right)^{1/2} \right], \tag{58}$$

$$L_1 = 0.5 \ln\left(1 + 9 \frac{Z_{eff} Z_0}{\Gamma^2} \left(\max\left\{ 1, \left[\frac{Z_{eff}}{3\Gamma\left(1+Z_0\right)} \right]^{1/2} \right\} \right)^2 \right), \tag{59}$$

where Γ is the parameter of nonideality:

$$\Gamma = \frac{Z_{eff} Z_0}{r_0} \min\left(\frac{1}{\theta}, \frac{1}{\theta_F} \right). \tag{60}$$

The next approximation is used for effective charge:

$$Z_{eff} = \max\left\{1, Z_0 + 0.5 / Z_0\right\}, \tag{61}$$

where Z_0 is ion average charge in accordance with the Hartree-Fock-Slater model [23].

Equations (1), (2), (3), (15), (16) (or (52)), (17) (or (39), or (51)), (31), (32), (33), (53) coupled with the tabular equations of state and homogeneous initial conditions make up the Cauchy problem for kinetic, hydrodynamic and Maxwell equations. Problem is posed for the investigation of interaction between electron beam and polarizable and magnetizable substance.

The main correlations between electromagnetic and thermomechanical effects can be estimated without considering polarization and magnetization when $|\mathbf{v}| \ll c$. In addition, both of permittivities should not change visible at scales of electron stopping path and characteristic hydrodynamic time.

The following simplified equations should be considered for nonpolarized and nonmagnetized substance instead of (16), (17):

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \left(\mathbf{v}\nabla\right)\mathbf{v} \right) = -\mathrm{grad} p + \mathbf{F}_l + \mathbf{Q}^p, \tag{62}$$

$$\frac{\partial}{\partial t}\left(\rho\left(\frac{v^2}{2} + u_0 \right) \right) = -\mathrm{div}\left(\mathbf{v}\rho\left(\frac{v^2}{2} + u_0 \right) + \mathbf{v}p \right) + A + Q^{\varepsilon}, \tag{63}$$

where $\mathbf{F}_l$ is the Lorentz force affecting the ionized substance, A is the power of the specific total electromagnetic field work.

$$\mathbf{F}_l = q\left(\mathbf{E} + \frac{1}{c}\left[\mathbf{v}\times\mathbf{B}\right] \right) + \frac{1}{c}\left[\mathbf{j}^*\times\mathbf{B}\right], \tag{64}$$

$$A = q\mathbf{E}\mathbf{v} + \mathbf{j}^*\mathbf{E} = \left(\mathbf{j} - \mathbf{j}_{ext}\right)\mathbf{E}. \tag{65}$$

Let's define the energy A_H which field spends on substance heating:

$$A_H = A - \mathbf{F}_l \mathbf{v} = \mathbf{j}^* \mathbf{E} - \frac{1}{c}\left(\mathbf{v}\left[\mathbf{j}^* \times \mathbf{B}\right]\right). \tag{66}$$

Relations (64), (65), (66) determine the electromagnetic field influence on ionized substance. Substituting (53) to (64) and (66) one can receive the expressions for Lorentz force and the heating power density:

$$\mathbf{F}_l = q\left(\mathbf{E} + \frac{1}{c}\left[\mathbf{v} \times \mathbf{B}\right]\right) + \frac{\sigma}{c}\left[\mathbf{E} \times \mathbf{B}\right] + \frac{\sigma}{c^2}\left(\mathbf{v}B^2 - \mathbf{B}\left(\mathbf{vB}\right)\right), \tag{67}$$

$$A_H = \sigma E^2 - 2\frac{\sigma}{c}\left(\mathbf{v}\left[\mathbf{E} \times \mathbf{B}\right]\right) - \frac{\sigma}{c^2}\left(v^2 B^2 - \left(\mathbf{vB}\right)^2\right). \tag{68}$$

Total medium heating by the field, electrons, positrons and photons is equal to:

$$\widetilde{A}_H = A_H + Q^{\varepsilon} - \mathbf{v}\mathbf{Q}^p. \tag{69}$$

The Coulomb law (33) enables to express the Lorentz force through the electromagnetic field components only:

$$\mathbf{F}_l = \left(\mathbf{E} + \frac{1}{c}\left[\mathbf{v} \times \mathbf{B}\right]\right)\left(\frac{\operatorname{div}\mathbf{D}}{4\pi} - q_{ext}\right) + \frac{\sigma}{c}\left[\mathbf{E} \times \mathbf{B}\right] + \frac{\sigma}{c^2}\left(\mathbf{v}B^2 - \mathbf{B}\left(\mathbf{vB}\right)\right). \tag{70}$$

3. NUMERICAL ALGORITHMS

The numerical algorithm for hydrodynamic equation is based on the conservative finite-difference scheme of the increased order of accuracy [25]. It is the Kolgan scheme which is modified and generalized for 3D unstructured grids. The predictor-corrector scheme integrates grid equations with respect to time variable. Since the scheme is explicit, the Courant–Friedrichs–Lewy condition should be implemented. The finite-difference scheme for Maxwell equations is represented in [20]. It is also explicit; it provides the second order of accuracy on homogeneous Cartesian grid, implements the discrete analogue of the energy conservation law.

Let's consider the scheme for the system of electromagnetic and hydrodynamic equations. It is necessary to construct finite-difference analogues for Lorentz force (70) and heating (68) which provide conservativeness of the general scheme.

Following relations [20] determine computational grid for Maxwell equations (31), (32), (33), (53) with respect to variable x:

$$x_{i+1} = x_i + \Delta_i \ , \ i = 0, \ldots, N_x - 1 \ , \ x_0 = x_{\min} \ , \ x_{N_x} = x_{\max} \ ,$$
$$x_{i+1/2} = (x_i + x_{i+1})/2 \ , \ i = 0, \ldots, N_x - 1 \ , \ x_{-1/2} = x_0 , \ x_{N_x+1/2} = x_{N_x} ,$$
$$\delta_i = x_{i+1/2} - x_{i-1/2} \ , \ i = 0, \ldots, N_x , \ \delta_0 = \Delta_0 / 2 \ , \ \delta_0 = \Delta_0 / 2 , \ \delta_{N_x} = \Delta_{N_x - 1} / 2 .$$

Computational grids with respect to variables y, z are introduced in a similar manner. Grid parameters provide the localization of coefficients discontinuity on surfaces $x = x_i$, $y = y_j$, $z = z_k$. The coefficients values are defined at the grid points with half-integer spatial indexes. These points coincide with centers of rectangular parallelepipeds which are organized by

intersection of planes $x = x_i, x_{i+1}$, $y = y_j, y_{j+1}$, $z = z_k, z_{k+1}$. The current density and electric field components are defined in the centers of corresponding parallel parallelepiped edges. The magnetic field components are defined in the centers of those parallelepiped faces, to which they are normal. The convective current and medium moving are not under consideration in [20].

Time grid consists of points t_n with intervals $\tau_n = t_{n+1/2} - t_{n-1/2};\ n = 2,..., N_t - 1$. The Grid functions E^x, E^y, E^z are defined in the integer moments of time t_n, H^x, H^y, H^z are defined at the half-integer points $t_{n+1/2}$.

Let's consider grid analogous for Maxwell equations (31), (32) [20]:

$$\left(H^{z\,n+1/2}_{i+1/2,j+1/2,k} - H^{z\,n+1/2}_{i+1/2,j-1/2,k}\right)\Big/\delta y_j - \left(H^{y\,n+1/2}_{i+1/2,j,k+1/2} - H^{y\,n+1/2}_{i+1/2,j,k-1/2}\right)\Big/\delta z_k =$$
$$= \left(D^{x\,n+1}_{i+1/2,j,k} - D^{x\,n}_{i+1/2,j,k}\right)\Big/c\tau_{n+1/2} + \frac{4\pi}{c}\overset{\circ}{I}{}^{x}_{i+1/2,j,k}.$$
$$\left(H^{x\,n+1/2}_{i,j+1/2,k+1/2} - H^{x\,n+1/2}_{i,j+1/2,k-1/2}\right)\Big/\delta z_k - \left(H^{z\,n+1/2}_{i+1/2,j+1/2,k} - H^{z\,n+1/2}_{i-1/2,j+1/2,k}\right)\Big/\delta x_i =$$
$$= \left(D^{y\,n+1}_{i,j+1/2,k} - D^{y\,n}_{i,j+1/2,k}\right)\Big/c\tau_{n+1/2} + \frac{4\pi}{c}\overset{\circ}{I}{}^{y}_{i,j+1/2,k}.$$
$$\left(H^{yn+1/2}_{i+1/2,j,k+1/2} - H^{yn+1/2}_{i-1/2,j,k+1/2}\right)\Big/\delta x_i - \left(H^{xn+1/2}_{i,j+1/2,k+1/2} - H^{xn+1/2}_{i,j-1/2,k+1/2}\right)\Big/\delta y_j =$$
$$= \left(D^{z\,n+1}_{i,j,k+1/2} - D^{z\,n}_{i,j,k+1/2}\right)\Big/c\tau_{n+1/2} + \frac{4\pi}{c}\overset{\circ}{I}{}^{z}_{i,j,k+1/2}.$$
$$\left(E^{yn+1}_{i,j+1/2,k+1} - E^{yn+1}_{i,j+1/2,k}\right)\Big/\Delta z_k - \left(E^{zn+1}_{i,j+1,k+1/2} - E^{zn+1}_{i,j,k+1/2}\right)\Big/\Delta y_j =$$
$$= \left(H^{xn+3/2}_{i,j+1/2,k+1/2} - H^{xn+1/2}_{i,j+1/2,k+1/2}\right)\Big/c\tau_{n+1}.$$
$$\left(E^{zn+1}_{i+1,j,k+1/2} - E^{zn+1}_{i,j,k+1/2}\right)\Big/\Delta x_i - \left(E^{xn+1}_{i+1/2,j,k+1} - E^{xn+1}_{i+1/2,j,k}\right)\Big/\Delta z_k =$$
$$= \left(H^{yn+3/2}_{i+1/2,j,k+1/2} - H^{yn+1/2}_{i+1/2,j,k+1/2}\right)\Big/c\tau_{n+1}.$$
$$\left(E^{xn+1}_{i+1/2,j+1,k} - E^{xn+1}_{i+1/2,j,k}\right)\Big/\Delta y_j - \left(E^{yn+1}_{i+1,j+1/2,k} - E^{yn+1}_{i,j+1/2,k}\right)\Big/\Delta x_i =$$
$$= \left(H^{zn+3/2}_{i+1/2,j+1/2,k} - H^{zn+1/2}_{i+1/2,j+1/2,k}\right)\Big/c\tau_{n+1}.$$

The following designations will be used. Symbol s^n signifies function value s on time layer with number n. Symbol $s^{\circ}_{i+1/2,j,k}$ signifies the following weighted value the of grid function u:

$$s^{\circ}_{i+1/2,j,k} = u_{i+1/2,j-1/2,k-1/2}\frac{\Delta_{j-1}}{2\delta_j}\frac{\Delta_{k-1}}{2\delta_k} +$$
$$+ s_{i+1/2,j-1/2,k+1/2}\frac{\Delta_{j-1}}{2\delta_j}\frac{\Delta_k}{2\delta_k} + s_{i+1/2,j+1/2,k+1/2}\frac{\Delta_j}{2\delta_j}\frac{\Delta_k}{2\delta_k} + s_{i+1/2,j+1/2,k-1/2}\frac{\Delta_j}{2\delta_j}\frac{\Delta_{k-1}}{2\delta_k}.$$

The induction vector is determined as $D^x_{i+1/2,j,k} = \varepsilon^{\circ}_{i+1/2,j,k} E^x_{i+1/2,j,k}$, the total current density is denoted as $I^{\circ x}_{i+1/2,j,k} = \sigma^{\circ}_{i+1/2,j,k} E^x_{i+1/2,j,k} + J^x_{i+1/2,j,k}$, where $J^x_{i+1/2,j,k}$ is the extrinsic current density.

The following finite-difference law of electromagnetic field energy conservation [20] takes place in computational domain:

$$\left(W^{fd}(\mathrm{E}^{n+1},\mathrm{H}^{n+1}) - \mathrm{W}^{fd}(\mathrm{E}^{n},\mathrm{H}^{n})\right)\Big/\tau_n + Q^{fd} + A^{fd} + K^{fd} + S^{fd} = 0, \tag{71}$$

where $\mathrm{W}^{fd}(\mathrm{E}^n,\mathrm{H}^n)$ is the quadratic form composed from values of the electric and the magnetic fields strength components at time layer with number n. This quadratic form is a grid analogue of electromagnetic field energy. Finite-difference analogues of all current works express values A^{fd}, Q^{fd} and K^{fd}. The value S^{fd} expresses energy flow through the boundary domain.

The expression for finite-difference work of the extrinsic current in computational domain is:

$$A^{fd} = 4\pi \sum_{i=0}^{N_x-1}\sum_{j=0}^{N_y}\sum_{k=0}^{N_z} J^x_{i+1/2,\mathrm{j},\mathrm{k}} E^x_{i+1/2,\mathrm{j},\mathrm{k}} \Delta_i \delta_j \delta_k + 4\pi \sum_{i=0}^{N_x}\sum_{j=0}^{N_y-1}\sum_{k=0}^{N_z} J^y_{i,\mathrm{j}+1/2,\mathrm{k}} E^y_{i,\mathrm{j}+1/2,\mathrm{k}} \delta_i \Delta_j \delta_k + \tag{72}$$
$$+4\pi \sum_{i=0}^{N_x}\sum_{j=0}^{N_y}\sum_{k=0}^{N_z-1} J^z_{i,\mathrm{j},\mathrm{k}+1/2} E^z_{i,\mathrm{j},\mathrm{k}+1/2} \delta_i \delta_j \Delta_k,$$

where $E^x_{i+1/2,j,k}, J^x_{i+1/2,j,k}$ are the electric field strength and the current density at the point $x = x_{i+1/2}, y = y_j, z = z_k$. The total current work is expressed analogously.

The finite-difference work of the extrinsic current in the cell with the center at the point $x = x_{i+1/2}, y = y_{j+1/2}, \ z = z_{k+1/2}$ is equal to:

$$A^{fd}_{cell} = \pi \Delta_i \Delta_j \Delta_k \left(J^x_{i+1/2,\mathrm{j},\mathrm{k}} E^x_{i+1/2,\mathrm{j},\mathrm{k}} + J^x_{i+1/2,\mathrm{j}+1,\mathrm{k}} E^x_{i+1/2,\mathrm{j}+1,\mathrm{k}} + J^x_{i+1/2,\mathrm{j},\mathrm{k}+1} E^x_{i+1/2,\mathrm{j},\mathrm{k}+1} + J^x_{i+1/2,\mathrm{j}+1,\mathrm{k}+1} E^x_{i+1/2,\mathrm{j}+1,\mathrm{k}+1} + \right.$$
$$+ J^y_{i,\mathrm{j}+1/2,\mathrm{k}} E^y_{i,\mathrm{j}+1/2,\mathrm{k}} + J^y_{i+1,\mathrm{j}+1/2,\mathrm{k}} E^y_{i+1,\mathrm{j}+1/2,\mathrm{k}} + J^y_{i,\mathrm{j}+1/2,\mathrm{k}+1} E^y_{i,\mathrm{j}+1/2,\mathrm{k}+1} + J^y_{i+1,\mathrm{j}+1/2,\mathrm{k}+1} E^y_{i+1,\mathrm{j}+1/2,\mathrm{k}+1} + \tag{73}$$
$$\left. + J^z_{i,\mathrm{j},\mathrm{k}+1/2} E^z_{i,\mathrm{j},\mathrm{k}+1/2} + J^z_{i+1,\mathrm{j},\mathrm{k}+1/2} E^z_{i+1,\mathrm{j},\mathrm{k}+1/2} + J^z_{i,\mathrm{j}+1,\mathrm{k}+1/2} E^z_{i,\mathrm{j}+1,\mathrm{k}+1/2} + J^z_{i+1,\mathrm{j}+1,\mathrm{k}+1/2} E^z_{i+1,\mathrm{j}+1,\mathrm{k}+1/2} \right).$$

The finite-difference work of the conductive current in the computational domain is a sum of cell works in edges:

$$Q^{fd}_{cell} = \pi \Delta_i \Delta_j \Delta_k \left(J^x_{c\,i+1/2,j,k} E^x_{i+1/2,\mathrm{j},\mathrm{k}} + J^x_{c\,i+1/2,\mathrm{j}+1,\mathrm{k}} E^x_{i+1/2,\mathrm{j}+1,\mathrm{k}} + J^x_{c\,i+1/2,\mathrm{j},\mathrm{k}+1} E^x_{i+1/2,\mathrm{j},\mathrm{k}+1} + \right.$$
$$+ J^x_{c\,i+1/2,\mathrm{j}+1,\mathrm{k}+1} E^x_{i+1/2,\mathrm{j}+1,\mathrm{k}+1} + J^y_{c\,i,\mathrm{j}+1/2,\mathrm{k}} E^y_{i,\mathrm{j}+1/2,\mathrm{k}} + J^y_{c\,i+1,\mathrm{j}+1/2,\mathrm{k}} E^y_{i+1,\mathrm{j}+1/2,\mathrm{k}} + J^y_{c\,i,\mathrm{j}+1/2,\mathrm{k}+1} E^y_{i,\mathrm{j}+1/2,\mathrm{k}+1} + \tag{74}$$
$$+ J^y_{c\,i+1,\mathrm{j}+1/2,\mathrm{k}+1} E^y_{i+1,\mathrm{j}+1/2,\mathrm{k}+1} + J^z_{c\,i,\mathrm{j},\mathrm{k}+1/2} E^z_{i,\mathrm{j},\mathrm{k}+1/2} + J^z_{c\,i+1,\mathrm{j},\mathrm{k}+1/2} E^z_{i+1,\mathrm{j},\mathrm{k}+1/2} + J^z_{c\,i,\mathrm{j}+1,\mathrm{k}+1/2} E^z_{i,\mathrm{j}+1,\mathrm{k}+1/2} +$$
$$\left. + J^z_{c\,i+1,\mathrm{j}+1,\mathrm{k}+1/2} E^z_{i+1,\mathrm{j}+1,\mathrm{k}+1/2} \right),$$

where:

$$J^x_{c\,i+1/2,j,k} = \sigma^{\circ}_{i+1/2,j,k}\left(E^x_{i+1/2,j,k} + \frac{1}{c}\left[\mathbf{v}_{i+1/2,j,k} \times \mathbf{B}_{i+1/2,j+1/2,k+1/2}\right]^{\circ x}\right). \quad (75)$$

Let the substance velocity in the hydrodynamic finite-difference scheme be defined in cell centers. The charge density is defined in cell centers too. We define convective current density in edge center as:

$$J^x_{i+1/2,j,k} = \frac{1}{4\delta_j\delta_k}\Big(q_{i+1/2,j+1/2,k+1/2}v^x_{i+1/2,j+1/2,k+1/2}\Delta_j\Delta_k + q_{i+1/2,j-1/2,k+1/2}v^x_{i+1/2,j-1/2,k+1/2}\Delta_{j-1}\Delta_k + \\ + q_{i+1/2,j+1/2,k-1/2}v^x_{i+1/2,j+1/2,k-1/2}\Delta_j\Delta_{k-1} + q_{i+1/2,j-1/2,k-1/2}v^x_{i+1/2,j-1/2,k-1/2}\Delta_{j-1}\Delta_{k-1}\Big) = \overset{\circ}{J}{}^x_{i+1/2,j,k}, \quad (76)$$

where $J^x_{i+1/2,j,k}$, $q_{i+1/2,j+1/2,k+1/2}$, $v^x_{i+1/2,j+1/2,k+1/2}$ are the x-component of convective current density, the charge density and the substance speed in cell center.

The work K^{fd} и K^{fd}_{cell} of the convective current, expressed by (76), is defined by analogy with the extrinsic one (72), (73).

If dielectric permittivity is constant, only convective and conductive currents fulfils the energy exchange between substance and field in accordance with (34), (65).

Let's define specific Lorentz force component for the cell with center at the point $x = x_{i+1/2}, y = y_{j+1/2}, z = z_{k+1/2}$ according to expression (67). It is necessary to interpolate values of the electromagnetic field strength components in the cell center. We shall calculate the magnetic field as an average value with respect to all cell faces with weight δ_i. The Electric field will be calculated with respect to all cell edges with weight $\delta_i\delta_j$:

$$\mathbf{F}^l_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2} = q_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}\left(\mathbf{E}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2} + \frac{1}{c}\left[\mathbf{v}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2} \times \mathbf{B}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}\right]\right) + \\ + \frac{\sigma_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}}{c}\left[\mathbf{E}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2} \times \mathbf{B}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}\right] + \\ + \frac{\sigma_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}}{c^2}\mathbf{v}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}B^2_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2} - \\ - \frac{\sigma_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}}{c^2}\mathbf{B}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}\left(\mathbf{v}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}\mathbf{B}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}\right), \quad (77)$$

where $\mathbf{F}^l_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}$ is the finite-different analogue of the specific Lorentz force.

The work of convective and conductive currents doesn't appear in the algorithm explicitly. So, it is necessary to construct the finite-difference analogue of work, which is expended on heating only, in other words, on change of the internal energy in cell. To do this, let's multiply Lorentz force by hydrodynamic velocity scalarly. Then let's subtract the result from the total work of convective and conductive currents:

$$A^H_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2} = Q^{fd}_{cell} - K^{fd}_{cell} - \mathbf{E}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}\mathbf{v}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}q_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2} - \\ - \frac{\sigma}{c}\left(\mathbf{v}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}\left[\mathbf{E}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2} \times \mathbf{B}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}\right]\right) - \quad (78)$$

$$-\frac{\sigma}{c^2}\left(v^2_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}B^2_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}-\left(\mathbf{v}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}\mathbf{B}_{i+1/2,\mathrm{j}+1/2,\mathrm{k}+1/2}\right)^2\right).$$

Finally, it's necessary to express charge density in field components. The definition of total charge density at the grid point follows from the finite-different scheme [20]:

$$q^{tot}_{i,j,k}=\frac{1}{4\pi}\left(\frac{D_{i+1/2,\mathrm{j},\mathrm{k}}-D_{i-1/2,\mathrm{j},\mathrm{k}}}{\delta_i}+\frac{D_{i,\mathrm{j}+1/2,\mathrm{k}}-D_{i,\mathrm{j}-1/2,\mathrm{k}}}{\delta_j}+\frac{D_{i,\mathrm{j},\mathrm{k}+1/2}-D_{i,\mathrm{j},\mathrm{k}-1/2}}{\delta_k}\right), \tag{79}$$

where $D_{i+1/2,j,k}$ is the component of electric displacement. It is defined in the center of cell edge. The edge is parallel to the component. We define the charge density as a difference between total charge averaged with all cell points with weight $\delta_i\delta_j\delta_k$ and the charge density of fast electrons q^{ext}:

$$\begin{aligned}q_{i+1/2,j+1/2,k+1/2}=&\frac{1}{8V_{i+1/2,j+1/2,k+1/2}}\left(q^{tot}{}_{i,j,k}\delta_i\delta_j\delta_k+q^{tot}{}_{i+1,j,k}\delta_{i+1}\delta_j\delta_k+q^{tot}{}_{i,j+1,k}\delta_i\delta_{j+1}\delta_k+\right.\\&+q^{tot}{}_{i,j,k+1}\delta_i\delta_j\delta_{k+1}+q^{tot}{}_{i+1,j+1,k}\delta_{i+1}\delta_{j+1}\delta_k+q^{tot}{}_{i+1,j,k+1}\delta_{i+1}\delta_j\delta_{k+1}+q^{tot}{}_{i,j+1,k+1}\delta_i\delta_{j+1}\delta_{k+1}+\\&\left.+q^{tot}{}_{i+1,j+1,k+1}\delta_{i+1}\delta_{j+1}\delta_{k+1}+q^{tot}{}_{i+1,j+1,k+1}\delta_{i+1}\delta_{j+1}\delta_{k+1}\right)-q^{ext}{}_{i+1/2,j+1/2,k+1/2},\end{aligned} \tag{80}$$

where $V_{i+1/2,j+1/2,k+1/2}$ is cell volume:

$$\begin{aligned}V_{i+1/2,j+1/2,k+1/2}=&\frac{1}{8}\left(\delta_i\delta_j\delta_k+\delta_{i+1}\delta_j\delta_k+\delta_i\delta_{j+1}\delta_k+\delta_i\delta_j\delta_{k+1}+\right.\\&\left.+\delta_{i+1}\delta_{j+1}\delta_k+\delta_{i+1}\delta_j\delta_{k+1}+\delta_i\delta_{j+1}\delta_{k+1}+\delta_{i+1}\delta_{j+1}\delta_{k+1}\right).\end{aligned} \tag{81}$$

Let's substitute (80) in (77), (78). In this way, we express the Lorentz force and substitute heating in terms of electromagnetic field and hydrodynamic velocity. They can be determined by solving Maxwell and hydrodynamic equations.

Formulae (77) and (78) express the finite-difference momentum and the internal energy. They are transmitted in the hydrodynamic finite-difference scheme as a source. It provides the conservativeness of the finite-difference scheme in general.

Now let's consider the finite-difference hydrodynamic equations. Grid functions in these equations are attributed to cell centers and are described in [25]. The transition to the next time level is performed in two stages. At the first stage pressure force is considered. The intermediate values of the substance velocity $\tilde{\mathbf{v}}$ and the specific energy density $\tilde{w}$ are calculated:

$$\begin{aligned}&\frac{\rho^n_{i+1/2,j+1/2,k+1/2}\tilde{\mathbf{v}}^n_{i+1/2,j+1/2,k+1/2}-\rho^n_{i+1/2,j+1/2,k+1/2}\mathbf{v}^n_{i+1/2,j+1/2,k+1/2}}{\tau_n}=-\frac{1}{V_{i+1/2,j+1/2,k+1/2}}\cdot\\&\cdot\left(F_{i+1,j+1/2,k+1/2}-F_{i,j+1/2,k+1/2}+F_{i+1/2,j+1,k+1/2}-F_{i+1/2,j,k+1/2}+F_{i+1/2,j+1/2,k+1}-F_{i+1/2,j+1/2,k}\right),\end{aligned} \tag{82}$$

$$\frac{\rho^n_{i+1/2,j+1/2,k+1/2}\tilde{w}^n_{i+1/2,j+1/2,k+1/2}-\rho^n_{i+1/2,j+1/2,k+1/2}w^n_{i+1/2,j+1/2,k+1/2}}{\tau_n}=-\frac{1}{V_{i+1/2,j+1/2,k+1/2}}\cdot \tag{83}$$

$$\cdot\left(A_{i+1,j+1/2,k+1/2}-A_{i,j+1/2,k+1/2}+A_{i+1/2,j+1,k+1/2}-A_{i+1/2,j,k+1/2}+A_{i+1/2,j+1/2,k+1}-A_{i+1/2,j+1/2,k}\right),$$

where $V_{i+1/2,j+1/2,k+1/2}$ is the volume of the cell with the center $i+1/2, j+1/2, k+1/2$, F is the force, acting on a face, A is the work of the force F.

$$w_{i+1/2,j+1/2,k+1/2}=u_{i+1/2,j+1/2,k+1/2}+v^2_{i+1/2,j+1/2,k+1/2}/2, \tag{84}$$

where $u_{i+1/2,j+1/2,k+1/2}$ is the internal substance energy.

Force and work are expressed in terms of velocity and pressure in the following way:

$$\mathbf{F}=P\cdot\mathbf{S}, \tag{85}$$

$$A=\left(\mathbf{S}\cdot\mathbf{v}\right)P, \tag{86}$$

where $\mathbf{S}$ is the area of the oriented face, which is directed to the cell with next number, P is the pressure in the cell center. Velocity $\mathbf{v}$ is defined in the face center.

Convective transport is considered at the second stage. The Final values ρ^{n+1}, $\mathbf{v}^{n+1}$, w^{n+1} on the upper time layer are calculated:

$$\frac{\rho^{n+1}_{i+1/2,j+1/2,k+1/2}-\rho^{n}_{i+1/2,j+1/2,k+1/2}}{\tau_n}=-\frac{1}{V_{i+1/2,j+1/2,k+1/2}}\left(F^{\rho}_{i+1,j+1/2,k+1/2}-\right.$$
$$\left.-F^{\rho}_{i,j+1/2,k+1/2}+F^{\rho}_{i+1/2,j+1,k+1/2}-F^{\rho}_{i+1/2,j,k+1/2}+F^{\rho}_{i+1/2,j+1/2,k+1}-F^{\rho}_{i+1/2,j+1/2,k}\right), \tag{87}$$

$$\frac{\rho^{n+1}_{i+1/2,j+1/2,k+1/2}\mathbf{v}^{n+1}_{i+1/2,j+1/2,k+1/2}-\rho^{n}_{i+1/2,j+1/2,k+1/2}\tilde{\mathbf{v}}^{n}_{i+1/2,j+1/2,k+1/2}}{\tau_n}=-\frac{1}{V_{i+1/2,j+1/2,k+1/2}}\times$$
$$\times\left(\mathbf{F}^{\rho\mathbf{v}}_{i+1,j+1/2,k+1/2}-\mathbf{F}^{\rho\mathbf{v}}_{i,j+1/2,k+1/2}+\mathbf{F}^{\rho\mathbf{v}}_{i+1/2,j+1,k+1/2}-\mathbf{F}^{\rho\mathbf{v}}_{i+1/2,j,k+1/2}+\mathbf{F}^{\rho\mathbf{v}}_{i+1/2,j+1/2,k+1}-\mathbf{F}^{\rho\mathbf{v}}_{i+1/2,j+1/2,k}\right), \tag{88}$$

$$\frac{\rho^{n+1}_{i+1/2,j+1/2,k+1/2}w^{n+1}_{i+1/2,j+1/2,k+1/2}-\rho^{n}_{i+1/2,j+1/2,k+1/2}\tilde{w}^{n}_{i+1/2,j+1/2,k+1/2}}{\tau_n}=-\frac{1}{V_{i+1/2,j+1/2,k+1/2}}\times$$
$$\times\left(A^{\rho w}_{i+1,j+1/2,k+1/2}-A^{\rho w}_{i,j+1/2,k+1/2}+A^{\rho w}_{i+1/2,j+1,k+1/2}-A^{\rho w}_{i+1/2,j,k+1/2}+A^{\rho w}_{i+1/2,j+1/2,k+1}-A^{\rho w}_{i+1/2,j+1/2,k}\right), \tag{89}$$

where F^{ρ}, $\mathbf{F}^{\rho\mathbf{v}}$, $F^{\rho w}$ are the convective flows of ρ, $\rho\mathbf{v}$, ρw through the corresponding face:

$$F^{\rho}=\left(\mathbf{S}\cdot\tilde{\mathbf{v}}\right)\left(\rho_{upv}+\Delta\rho_{upv}\right), F^{\rho\mathbf{v}}=\left(\mathbf{S}\cdot\tilde{\mathbf{v}}\right)\left(\rho\mathbf{v}_{upv}+\Delta\rho\mathbf{v}_{upv}\right),$$
$$F^{\rho w}=\left(\mathbf{S}\cdot\tilde{\mathbf{v}}\right)\left(\rho w_{upv}+\Delta\rho w_{upv}\right). \tag{90}$$

Index upv means, that the value is defined for two cells which adjoin to face from which the velocity $\tilde{\mathbf{v}}$ is directed. The symbol Δ means an amendment to corresponding value, which provides monotony and increased accuracy order. Let cell with index $i+1/2, j+1/2, k+1/2$ is windward for one with the index $i+1/2, j+1/2, k$. Then the amendment will be equal to minimum modulus of two values:

$$\left|\Delta\rho_{upw}\right| = \min[\left|\rho_{i+1/2,j+1/2,k} - \rho_{i+1/2,j+1/2,k+1/2}\right|, \left|\rho_{i+1/2,j+1/2,k+1/2} - \rho_{i+1/2,j+1/2,k+1}\right|]. \quad (91)$$

The amendment for $\rho\mathbf{v}$ and ρw is calculated analogously. If differences in (91) have different signs, amendment is equal to zero.

The amendments to velocity and the internal energy conditioned by the Lorentz force and energy of substance heated by the electromagnetic field:

$$\Delta\mathbf{v}_{i+1/2,j+1/2,k+1/2} = \mathbf{F}^{l}_{i+1/2,j+1/2,k+1/2}\tau_n \Big/ \rho^{n+1}_{i+1/2,j+1/2,k+1/2}, \quad (92)$$

$$\Delta u_{i+1/2,j+1/2,k+1/2} = A^{H}_{i+1/2,j+1/2,k+1/2}\tau_n \Big/ \rho^{n+1}_{i+1/2,j+1/2,k+1/2}. \quad (93)$$

Both the electrodynamic and the hydrodynamic finite-difference schemes have the second order of accuracy. The general finite-difference scheme has the second order of accuracy too.

4. TEST CALCULATION

The effectiveness of the algorithm has been investigated in test calculation. Conditions were chosen in accordance with the experimental installation. The beam of electrons with energy of 200 keV falls normally on the epoxy resin barrier. The pulse duration was 150 ns, the electron fluence $N_e = 3 \cdot 10^{16}$ 1/cm^2, the beam cross-sectional area was 1 cm^2. Such experiments with electron beams are conducted at the National Research Center "Kurchatov Institute" on the accelerator CALAMARY [26].

Full 3D modelling of electron transport, substance dynamics and the electromagnetic field was carried out on cluster K-100 at Keldysh Institute of Applied Mathematics.

The energy release, current density and electromagnetic field distributions were calculated by REMP package.

Figure 1 represents the dependence of energy release on the coordinate normal to barrier surface. The coordinate origin is located on the irradiated barrier surface in the center of the beam cross section. The energy release reaches the value of 50 kJ/cm^3 and exceeds sublimation heat to a depth of 0.02 cm. Beyond this distance the energy release decreases sharply. One can see the regions of evaporation, melting and heating.

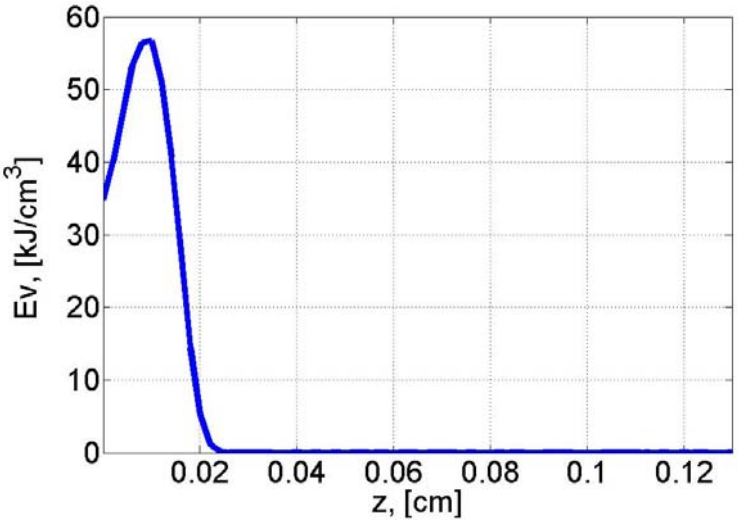

Fig. 1: Energy release as a function of normal coordinate

Figure 2 represents the dependence of the electric field strength on the time variable at a depth of 0.01 cm in barrier. The thermal and the radiation conductivities were calculated in the framework of the approximations considered in Section 2.

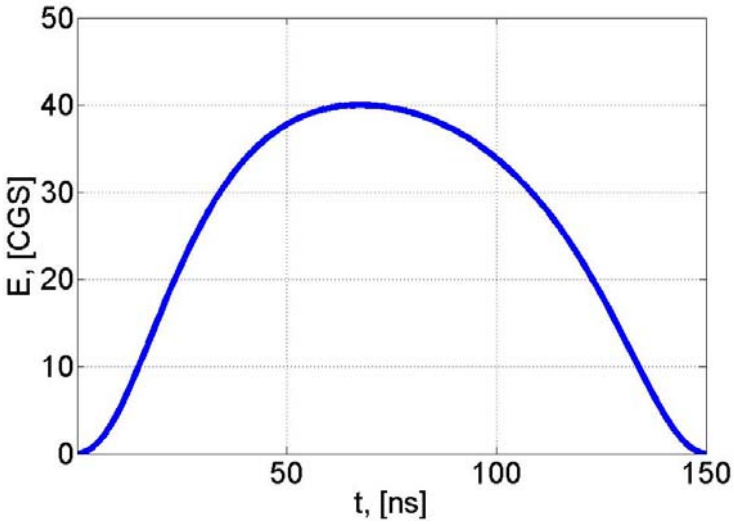

Fig. 2: The electric field strength (ESU CGS) as a function of time variable

The calculated distributions of energy release, current density and electromagnetic fields have been used for the simulation of thermomechanical effects by the MARPLE package. No perceptible dependence of their parameters on the electric field has been detected.

The simulation has brought to light the high sensitivity of results to the value of conductivity. It should be noted, that all models in Chapter 2 are founded on classic fundamental equations. Only both of conductivities are considered within the framework of the empirical model. It involves an inadmissible computational load required for the modeling of ionization spectrum degradation within the framework of kinetic theory. The finiteness of the degradation time leads to the delay in the development of conductivity relative to the electric current. As a result, electric field increases.

The next computer experiment was carried out with consideration of the fact that one can establish the majorant estimate of electric field equating conductivity to zero.

Figure 3 represents the dependence of the electric field strength on the normal coordinate for nonconducting barrier.

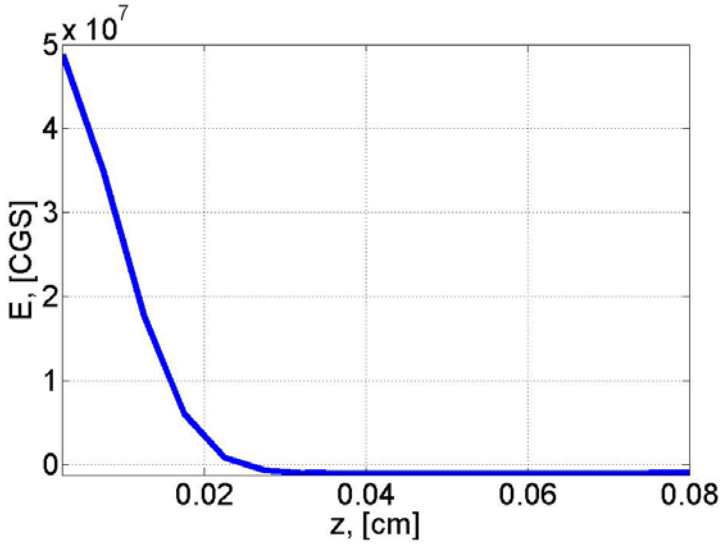

Fig. 3: The electric field strength (ESU CGS) as a function of the normal coordinate in nonconducting barrier

It's obvious that the electric field with the strength of the order of 10^7 SGSE can't be observed in a real experiment. This value exceeds significantly dielectric rigidity of all known substances. Nevertheless, let's consider the influence of such field on barrier substance dynamics.

Figure 4 represents the dependence of the following values on normal coordinate. The red lines show the dependence of substance density. The blue lines show the dependence of specified ~~speed~~ normal velocity component. Hereinafter solid lines refer to the calculation experiment in which electric field is taken into account. Dashed lines refer to the calculation experiment where the field influence is neglected. One can see, that electric field density peak value decreases from 2.5 to 2.3 g/cm^3 and the specified speed increases from 7 10^6 to 9 10^6 cm/s.

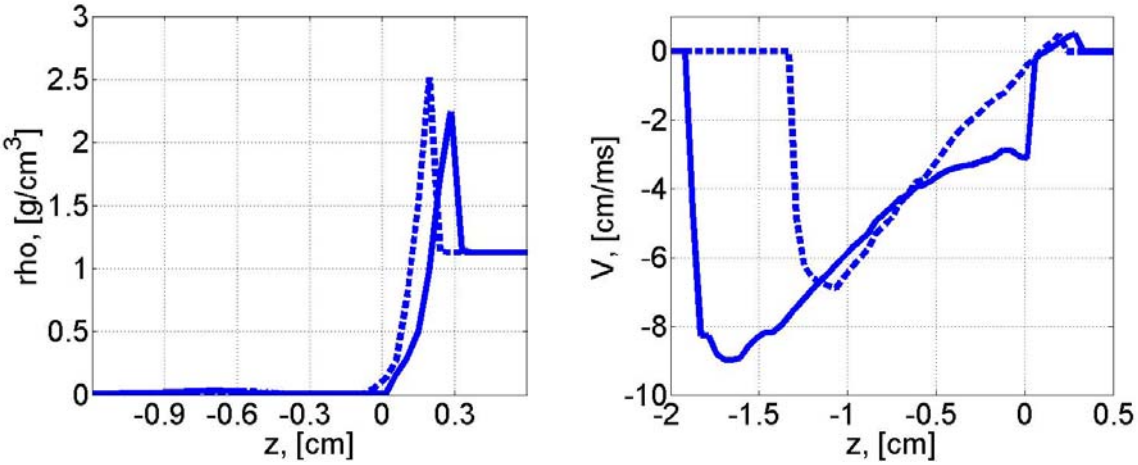

Fig. 4: The substance density and normal component of specified velocity as a function of normal coordinate. Solid lines – with electric field considering, dashed lines – with electric field neglecting

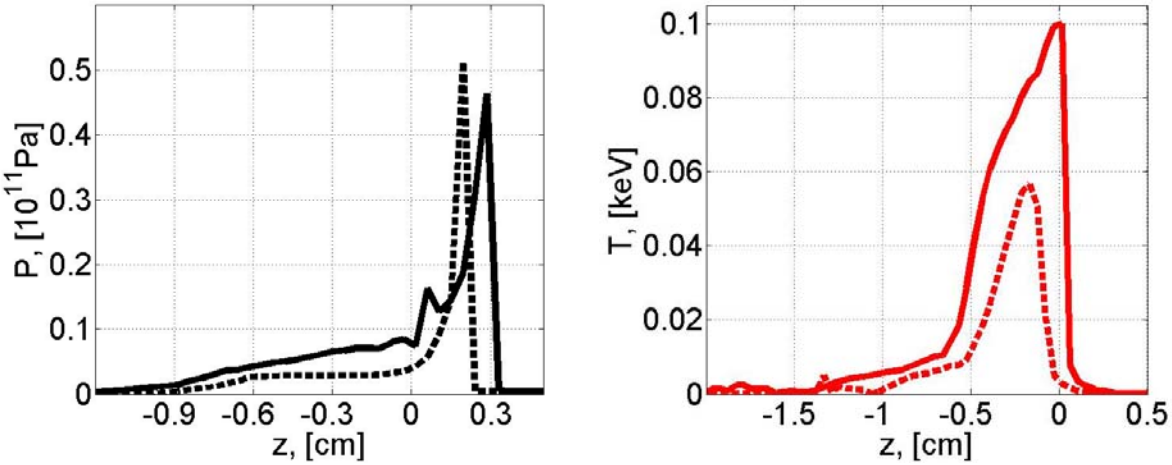

Fig. 5: The pressure and temperature as a function of normal coordinate. Solid lines – with electric field considering, dashed lines – with electric field neglected

Figure 5 shows the pressure peak value decrease from 0.08×10^{11} to 0.07×10^{11} Pa. The temperature increased from 0.05 to 0.1 keV.

The electric field of unrealistically large amplitude changed the thermodynamic parameters only by tens of percent.

5. CONCLUSIONS

Classis kinetic equations for electron, photon and positron distribution functions in coordinate-momentum phase space describe the radiation transport in barrier substance. Collision integrals model the medium impact ionization and excitation by electrons, elastic scattering and bremsstrahlung. Compton and coherent scattering, photoabsorption and pair production of bremsstrahlung photons complement the set of physical effects under consideration. Maxwell equations describe external and self-consistent electromagnetic fields. Euler equations model the substance dynamics under the influence of the electromagnetic field and energy release. Hydrodynamic consideration unites the computational domain regions of heating, melting and evaporation within the framework of the common thermodynamical mathematical model.

The kinetic equations are solved in the space of finite generalized functions. The power of energy release and electric current density are defined as a linear functional in this space. It enables the particle-in-cell method usage for kinetic equations numerical simulation [18] within the scope of REMP package.

Previously developed fully conservative finite difference schemes for Maxwell and hydrodynamic equations have formed the basis of numerical algorithm. The Finite-difference analogues of Joule heating, the Lorentz force and the convective current provide the implementation of finite difference energy conservation law for the system of the united electromagnetic field and ionized substance.

The interaction between thermodynamic and electromagnetic fields is investigated in the simulation of experiment with the electron beam of CALAMARY accelerator.

The energy release of the high-current electron is comparable with the Lorentz force in the nonconductive medium. The radiative and thermal conductivity of the barrier changes the

result of simulation dramatically. The energy release in such media repeatedly exceeds the Lorentz force in nonconductive medium. The investigation showed that the conductivity mathematical model should be the main subject of further researches.

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REFERENCES

[1] Vladimir E. Fortov, *Extreme States of Matter. High Energy Density Physics*, Springer Series in Material Science, (2016).

[2] P. V. Breslavskiy, A. V. Mazhukin and O. N. Koroleva, "Simulation of the Dynamics of Plasma Expansion, the Formation and Interaction of Shock and Heat Waves in the Gas at the Nanosecond Laser Irradiation", *Mathematica Montisnigri*, **33**, 5-24 (2015).

[3] V. I. Mazhukin, M. M. Demin and A. V. Shapranov, "High-speed Laser Ablation of Metal with Pico- and Subpicosecond Pulses", *Applied Surface Science*, **302**, 6–10 (2014).

[4] Yu. M. Milekhin, D. N. Sadovnichii and A. P. Tyutnev, Electrical effects in propellant compositions radiated by electrons. *Combustion, Explosion, and Shock Waves,* **43** (3), 538-548 (2007).

[5] D. N. Sadovnichii, A. P. Tyutnev and Yu. M. Milekhin, "Charge accumulation in polymethylmetacrylate radiated by high-energy electrons", *Khim. Vysok. Énerg.*, **39**(3), 183–189 (2005).

[6] M. E. Zhukovskiy, M. B. Markov, S. V. Podolyako, I. A. Tarakanov, R. V. Uskov, A. M. Chlenov and V. F. Zinchenko, "Researching the spectrum of bremsstrahlung generated by the RIUS-5 electron accelerator", *Mathematica Montisnigri*, **35**, 54-67 (2016).

[7] N. F. Mott, H. S. W. Massey, *The theory of atomic collisions*, Oxford: Clarendon Press, (1965).

[8] H. S. W. Massey, E. H. S. Burhop, *Electronic and Ionic Impact Phenomena*, Oxford: Clarendon Press, (1969).

[9] H. Davies, H.A. Bethe and L.C. Maximon, "Theory of bremsstrahlung and pair production. Integral cross section for pair production", *Phys. Rev.* **93**, 788-795 (1954).

[10] M. Gryzinski, "Classic Theory of Electronic and Ionic Inelastic Collisions", *Phys. Rev.* **115** (2), 374-383 (1959).

[11] Yong-Ki Kim, M. E. Rudd, "Theory for Ionization of Molecules by Electrons" *Phys. Rev.* **50**, 3954-3967 (1994).

[12] M. B. Markov, S. V. Parot'kin, "The kinetic model of radiation-induced gas conductivity" *Mathematical Models and Computer Simulations*, **3** (6), 712–722 (2011).

[13] A. V. Berezin, Yu. A. Volkov, M. B. Markov, I. A. Tarakanov, "The radiation-induced conductivity of silicon". *Mathematica Montisnigri*, **33**, 69-87 (2015).

[14] Ya. B. Zel'dovich, Yu. P. Raizer, *Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena*, Academic Press, New York, (1968).

[15] L. D. Landau, E. M. Lifshitz, *The Classical Theory of Fields, 4th Edition,* Butterworth–Heinemann, Vol. 2, (1975).

[16] W. Heitler, *The Quantum Theory of Radiation,* Oxford: Clarendon Press, (1954).

[17] V. A. Gasilov, A. S. Boldarev, S. V. Dyachenko, O. G. Olkhovskaya, E. L. Kartasheva, G. A. Bagdasarov, S. N. Boldyrev, I. V. Gasilova, M. S. Boyarov, V. A. Shmyrov, "Program Package MARPLE3D for Simulation of Pulsed Magnetically Driven Plasma Using High Performance Computing", *Math. Mod.*, **24** (1), 55-87 (2012).

[18] A. V. Berezin, A. S. Vorontsov, M. E. Zhukovskiy, M. B. Markov, S. V. Parot'kin, "Particle method for electrons in a scattering medium", *Computational Mathematics and Mathematical*

Physics, **55** (9), 1534–1546 (2015).
[19] G. E. Shilov, *Generalized Functions and Partial Differential Equations,* Gordon and Breach Science Publishers Inc., (1968).
[20] A. V. Berezin, A. A. Kriukov, B. D. Pliushchenkov, "The method of electromagnetic field with the given wavefront calculation", *Math. Mod.*, **23** (3), 109-126 (2011).
[21] A. N. Andrianov, A. V. Berezin, A. S. Vorontsov, K. N. Efimkin, M. B. Markov, "The radiational electromagnetic fields modeling at the multiprocessor computing systems", *Math. Mod.*, **20** (3), 98–114 (2008).
[22] L. D. Landau, E. M. Lifshitz, *Fluid Mechanics*, *2nd Edition*, Butterworth–Heinemann, Vol. 6, (1987).
[23] A. F. Nikiforov, V. G. Novikov, V. B. Uvarov, *Quantum-Statistical Models of Hot Dense Matter: Methods for Calculation Opacity and Equation of State*, Birkhauser, Basel-Boston-Berlin, (2005).
[24] S.R. de Groot, P. Mazur, *Non-equilibrium Thermodynamics*, Amsterdam: North-Holland Publ. Co., (1962).
[25] A. S. Boldarev, V. A. Gasilov, O. G. Olkhovskaya "On the Solution of Hyperbolic Equations using Unstructured Grids", *Math. Mod.*, **8** (3), 51-78 (1996).
[26] V. A. Demidov, V. P. Efremov, M. V. Ivkin, A. N. Meshcheryakov, V. A. Petrov, "Effect of intense energy fluxes on vacuum-tight rubber", *Technical Physics,* **48** (6), 787–792 (2003).

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